
Zbl 091.04401

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Additive properties of random sequences of positive integers. (In English)

Acta Arith. 6, 83-110 (1960). [0065-1036]

Let ξ_n $(n = 1, 2, ...)$ be a sequence of independent random variables such that $P(\xi_n = 1) = p_n$, $P(\xi_n = 0) = 1 - p_n$, $0 \leq p_n \leq 1$ and $\sum_1^\infty p_n = \infty$. Let $V_1 < V_2 < \cdots$ be the values of n for which $\xi_n = 1$. The sequence $\{V_k\}$ is called a random sequence of positive integers generated by the sequence $\{p_n\}$. The authors investigate sums of two or more random sequences and give various conditions under which such sums have almost certainly positive density. These conditions include among other the sum of two sequences for which $p_n = cn^{-\frac{1}{2}}$. In this case one has almost certainly $\lim_{k\to\infty} \frac{v_k}{k^2} = \frac{1}{4}c^2$. This result is in contrast to the fact that the set of integers which can be represented as a sum of two squares has density 0. The authors also investigate the distribution of the number $f(n)$ of representations of a number n in the sum of two random sequences and the density of those integers which have exactly r representations. In the cases considered this density is positive. If $\{V_n\}$ is a random sequence and $f(n)$ the number of representations of n in the form $V_n + V_e$, then $f(n)$ is in the limit normally distributed if the variance of $f(n)$ tends to infinity with n. The authors also consider the distribution of the differences of two random sequences.

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Classification:
11B34 Representation functions
11B83 Special sequences of integers and polynomials
60F99 Limit theorems (probability)