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Erdős, Pál; Hajnal, András
Some remarks on set theory. VII. (In English)
Acta Sci. Math. 21, 154-163 (1960). [0001-6969]

[Part VI: Zbl 078.04203]
For a family F of sets let $pF = \sup|X|$ $(X \in F)$. For a cardinal r the authors say that F has the property $A(r)$ if some set X of cardinality $< r$ intersects every member of F: one could write $F \in A(r)$. Analogously, $F \in A(q,r)$ if $F' \subset F$, $|F'| < q \Rightarrow F' \in A(r)$. Def.: (1) $[p,q,r] \to s$ means that every F such that $pF = p$, $F \in A(q,r)$ possesses the property $A(s)$ too. F possesses the property $B(t)$ if every $F' \subseteq F$ with $|F'| = t$ has a subfamily F'' of cardinality t such that $\bigcap F''$ is non empty. Let $p = p(F)$; $F \in B(p) \Rightarrow F \in A(p)$(Theorem 6); if moreover $p = \aleph_\alpha$ is singular and $F \in B(\aleph_{cf\alpha}$ then $F \in A(\aleph_\alpha)$ (Theorem 7).
If $p = \aleph_\alpha$ is singular, $r > \aleph_{\mathrm{cf}\alpha}$ $q \le p^+$ then $[p,q,r] \nrightarrow s$ for $s \le p$ (Theorem 1); for $r \le \aleph_{\mathrm{cf}\alpha}$, $q > \aleph_{\mathrm{cf}\alpha}$ one has $[p,q,r] \to p$ (Theorem 2). If $\aleph_\alpha$ is regular, then $[\aleph_{\alpha+n}, \aleph_{\alpha+n+1}, \aleph_\alpha] \to \aleph_\alpha$ (Theorem 5).
Problem 1. Does $\aleph_{\omega+1}^{\aleph_0} \le 2^{\aleph_0}$. $\aleph_{\omega+1}$ imply $[\aleph_{\omega+1}\aleph_{\omega+2}\aleph_1] \to \aleph_\omega$? There are also other results, concerning the relation (1), in particular implied by the general continuum hypothesis.

G.Kurepa

Classification:
05D10 Ramsey theory