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Erdős, Pál; Taylor, S.J.
On the set of points of convergence of a lacunary trigonometric series and the equidistribution properties of related sequences (In English)
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The paper is devoted to the determination of the dimension of the sets of Lebesgues measure zero which are the only sets of ordinary and absolute convergence for the series (*) $\sum_{k=1}^{\infty} \sin(n_k x + \mu_k)$ where $0 \le \mu_k \le 2\pi$ and (n_k) is an increasing sequence of integers satisfying the condition $t_k = n_{k+1}/n_k \ge \varrho > 1$. Some of the theorems proved are:
(1) If t_k is an integer for large values of k, and $t_k \to \infty$, then $\sum |\sin n_k x| < \infty$ on a set of x's having the power of the continuum. (2) If $n_k = k!$ and $0 < y < \pi$ or $\pi < y < 2\pi$, then the series $\sum |\sin(n_k x - y)| < \infty$ for no value of x. (3) If $\sum t_k^{-1} < \infty$, then $\sum |\sin(n_k x - y)| < \infty$ for every value of x in a set of power of continium. (4) If $\lambda > 0, \mu > 0, \varrho > 0$ are constants such that λk^p for every integer k, then the dimensions (Besicovitch) of the set of x's for which $\sum |\sin(n_k x - \mu_k)| < \infty$ is zero if $0 < \varrho < 1$ and $1 - 1/\varrho$ if $\varrho > 1$. (5) If $t_k \to \infty$ then $\sum |\sin(n_k x - \mu_k)| < \infty$ in a set of values of x of dimension 1. If (n_k) is an increasing sequence of integers, we denote the sets of values x for which $((n_k x))$, the fractional part of $n_k x$, is not equidistributed in $(0,1)$ by E. As an application the following theorems are proved: (6) E has zero Lebesgue measure. (7) There exists a finite constant C and an increasing sequence of integers (n_k) such that $n_{k+1} - n_k < C$ and such that E is not enumerable. (8) If (n_k) is an increasing sequence of integers such that $n_{k+1} - n_k < C$, then E has dimension zero. (9) If $n_k < Ck^{\varrho}$ $(k = 1, 2, ...)$ then E has dimension not greater than 1. — $1 - 1/\varrho$. (10) If $t_k \ge \varrho > 1$ then the set E of values has dimension 1.

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Classification:
11K06 General theory of distribution modulo 1
42A55 Lacunary series