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A problem concerning the zeros of a certain kind of holomorphic function in the unit disk (In English)

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Annular functions are those functions, $f(z)$, which are holomorphic in the open unit disk D and have the property that $\lim_{n\to\infty} \min_{z\in J_n} |f(z)| = \infty$, where the J_n are members of a sequence of Jordan curves in D satisfying the following conditions: J_n lies the interior of J_{n+1}; and given an $\varepsilon > 0$, there exists an $n_0(\varepsilon)$ such that for every $n > n_0$, J_n lies in the region $1-\varepsilon < |z| < 1$. If we denote by $Z(f)$ the set of zeros of f, and by $Z'(f)$ the set of limit points of $Z(f)$, then the problem posed by the authors is: if f is an annular function, does $Z'(f) = K$, where K is the unit circle. Two theorems are presented which give sufficient conditions on f so that $Z' = K$. For example.
Theorem 1: Let f be an annular function, and suppose that there exists an everywhere dense subset E of K such that every point of E is the end point of an asymptotic path of f, then $Z' = K$.
The final theorem of the paper contains an example of an annular function that does not satisfy these sufficient conditions and the authors conclude that the basic problem is not only unsolved, but that the avenue of approach suggested in the first two theorems cannot lead to its successful resolution.

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Classification:
30C15 Zeros of polynomials, etc. (one complex variable)