Zbl 148.28903
Erdős, Pál; Piranian, G.
Essential Hausdorff cores of sequences (In English)
J. Indian Math. Soc., n. Ser. 30, 93-115 (1966).

In order to refine the concept of the core of a complex sequence which is due to Knopp, the authors compactify E^2 by adjoining the circle Γ of points (∞, ϑ), $0 \leq \vartheta < 2\pi$ with the topology of the disk shaped plane $E^2 \cup \Gamma$. If $x = \{x_n\}$ is a sequence in the disk shaped plane the core $K(x)$ is the set of all $p \in E^2 \cup \Gamma$ such that each half plane containing p contains x_n for infinitely many n. A row-finite matrix A is called core shrinking in the space of all complex sequences if $K(As) \subseteq K(s)$ for each sequence s. Let T be a family of row-finite core shrinking matrices that commute in pairs under matrix multiplication and define the essential T-core of s as $K(T, s) = \bigcap_{A \in T} K(s)$.
Theorem: Let A and B be two row-finite core-shrinking matrices such that $AB = BA$ and let $S = \{I, A, A^2, ...\}$, $T = \{I, B, B^2, ...\}$, $U = \{I, AB, (AB)^2, ...\}$. Then for each sequence s we have $K(S, s) \cap K(T, s) \supseteq K(U, s)$. The inclusion symbol cannot in general be reversed. Since every two Hausdorff matrices commute the authors discuss core shrinking Hausdorff matrices and in particular the Cesàro and Hölder methods. Let H denote the family of regular Hausdorff matrices and H^+ the family of core- shrinking Hausdorff matrices. A sequence s is said to be summable to a finite complex σ by the collective Hausdorff method $\{H\}$ provided there exists a matrix $A \in H$ such that $As \to \sigma$; s is said to be summable to $\tau \in E^2 \cup \Gamma$ by the KH^+ method of essential Hausdorff cores provided $K(H^+, s)$ consists of τ. Among other results the following is true. Theorem: In the space of sequences whose essential Hausdorff core is bounded, the method of essential Hausdorff cores is stronger than the collective Hausdorff method and it is consistent with it.
The paper contains many results as well as extended bibliography and suggests some open problems.

D.Leviatan

Classification:
40G05 Traditional summability methods
40C05 Matrix methods in summability