Erdős, Pál; Katai, I.
On the sum $\sum d_4(n)$ (In English)
Acta Sci. Math. 30, 313-324 (1969). [0001-6969]

Let $d(n)$ denote the number of divisors of n, and $d_k(n)$ be the k-fold iterate of $d(n)$, i. e. $d_1(n) = d(n)$ and $d_k(n) = d(d_{k-1}(n))$ for $k \geq 2$. It was conjectured by Bellman and Shapiro that the relation

$$\sum_{n \leq k} d_k(n) = c_k(1 + o(1))x \log_k x$$

holds, where $\log_k$ denotes the k-fold iterate of logarithm function. This was proved previously for $k = 2$ by the authors independently, for $k = 3$ by Kátai. Here the authors prove the case $k = 4$. The cases $k \geq 5$ seem to be very difficult.

Classification:

11N37 Asymptotic results on arithmetic functions