Zbl 188.34102
Erdős, Pál; Katai, I.
On the growth of $d_k(n)$ (In English)
Fibonacci Q. 7, 267-274 (1969). [0015-0517]

Let $d_k(n)$ denote the k-fold iterated $d(n)$, where $d(n)$ the number of divisors of n. Let l_k be the k-th element of the Fibonacci sequence ($l_{-1} = 0, l_0 = 1, l_k = l_{k-1} + l_{k-2}, k \geq 2$). We prove $d_k(n) < \exp(\log n)^{1/l_k+\varepsilon}$ for all fixed k, all positive ε and all sufficiently large values of n; further for every $\varepsilon > 0$ $d_k(n) > \exp(\log n)^{1/l_{k-\varepsilon}}$ for an infinity of values of n. For $n > 1$ let $k(n)$ denote the smallest k for which $d_k(n) = 2$. We prove

$$0 < \limsup(k(n)/\log\log\log n) < \infty.$$

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11N56 Rate of growth of arithmetic functions

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