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Two combinatorial problems in group theory. (In English)
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Sequences of elements from (additive) abelian groups are studied. Conditions under which a nonempty subsequence has sum equal to the group identity 0 are established. For example, an n-sequence with exactly k distinct terms represents 0 if the group has order $g \leq n + \binom{k}{2}$ and $n \geq k\binom{k}{2}$.
The least number $f(k)$ of distinct partial sums is also considered, for the case of k-sequences of distinct elements such that no nonempty partial sum is equal to 0. For example, $2k - 1 \leq f(k) \leq [\frac{1}{2}k^2] + 1$.
In this paper a sequence is a selection of members of a set, possibly with repetitions, in which order is not important; elements are members of sets, and terms are members of sequences.
Definition. Let $*$ be a binary operation on a set A, and let $S = (a_i)_{i=1}^n$ be a sequence of elements from A. S will be said to represent the element $x \in A$ if (i) x is a term in S, or (ii) there exist $x, z \in A$ such that $x = y * z$, and y and z are represented by disjoint subsequences of S. (Clearly this notion extends to general algebras.)
In particular, if $\langle G, +\rangle$ is an abelian group and $S = (a_i)_{i=1}^n$ is a sequence of elements from G, then S represents $x \in G$ just if there exists a sequence $E = (\epsilon_i)_{i=1}^n$ of elements from $\{0, 1\}$, not all 0, such that $\sum_{i=1}^n \epsilon_i a_i = x$.
We resolve here some aspects of the following two related problems. (1) Under what circumstances does an n-sequence of elements from an abelian group represent the zero element? (2) If an n-sequence of distinct elements from an abelian group does not represent the zero element, how many elements does it represent?

Classification:
20K99 Abelian groups
05A10 Combinatorial functions
05A20 Combinatorial inequalities
05-02 Research monographs (combinatorics)
00A07 Problem books