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A theorem in the partition calculus. (In English)

Can. Math. Bull. 15, 501-505 (1972); corrigendum ibid. 17, 305 (1974).

It is shown that if $h < \omega$ and $\nu < \omega_1$ then $\omega^{1+\nu h} \to (2^h, \omega^{1+\nu})^2$, as known to the authors since 1959. The literature of this result and related results are discussed. The proof is by induction on h from a more comprehensive theorem. For each ordered set S, $tp\, S$ is the order type of S. Consider any order type α. α is right-AI if and only if $\alpha = \beta + \gamma$, $\gamma \neq 0$ implies that $\gamma \geq \alpha$ (AI abbreviates additively indecomposable). α is strongly indecomposable (briefly, SI) if and only if whenever $A = B \cup C$ and α is the type of A, B, or C has type $\geq \alpha$. For each order type β, β is strong if and only if whenever $tp\, B = \beta$ and $D \subset B$ there are $n < \omega$ and subsets $D_1, \dots, D_n$ of D such that $tp\, D_i$ is SI for $i = 1, \dots, n$ and such that for each $M \subset D$, if $tp(M \cap D_i) \not\geq tp\, D_i$ for $i = 1, \dots, n$ then $tp\, M \approx tp\, D$. For example, each ordinal number and its reverse are strong. The theorem proved is that for each SI and right-AI order type α and each strong denumerable order type β, if $2\ lek < \omega$ and $\alpha \to (k, \gamma)^2$ then $\alpha\beta \to (2k, \gamma \cup \omega\beta)^2$. (It is added in proof that *F. Galvin* has proved the same thing with "strong" deleted from the hypothesis on β, thus settling a conjecture of the writers.)

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Classification:
04A10 Ordinal and cardinal numbers; generalizations
05A17 Partitions of integres (combinatorics)
05A99 Classical combinatorial problems