Zbl 273.26001

Ash, J.Marshall; Erdős, Paul; Rubel, L.A.

Very slowly varying functions. (In English)

Aequationes Math. 10, 1-9 (1974). [0001-9054]

Let φ be a positive non-decreasing real valued function defined on $[0,\infty)$, and let f be any real valued function defined on $[0,\infty)$. We say that f is φ-slowly varying if $\varphi(x)[f(x+\alpha)-f(x)]\to 0$ as $x\to\infty$ for each α. We say that f is uniformly φ-slowly varying if $\sup\{\varphi(x)|f(x+\alpha)-f(x)|:\alpha\in I\}\to 0$ as $x\to\infty$ for every bounded interval I. We state here five theorems that will be proved later in a longer communication. We also pose one question that seems to be difficult. Theorem 1. If f is φ-slowly varying and if $\sum i/\varphi(n)<\infty$, then f tends to a finite or infinite limit at ∞. Theorem 2. If f is φ- slowly varying and measurable, then f is uniformly φ-slowly vayring. Theorem 3. Let f be φ-slowly varying and let $\beta(x)=\sum_{j=0}^{\infty}1/\varphi(x+j)$. If $\varphi(x)\beta(x)$ is bounded, then f must be uniformly φ-slowly varying. Theorem 4. Suppose that $\sum 1/\varphi(n)<\infty$ and that $\varphi(x+1)/\varphi(x)\to 1$ as $x\to\infty$. Then there exists a function f that is φ-slowly varying but not uniformly φ-slowly varying. Theorem 5. Let $\beta(x)$ be the function of Theorem 4, and suppose that $\varphi(x)\beta(x)$ is unbounded, but that $\varphi(x)\beta(x)=o(x)$ as $x\to\infty$. Then there exists a function f that is φ-slowly varying but not uniformly φ- slowly varying. Question. Does there exist a function f such that $x[f(x+\alpha)-f(x)]\to 0$ as $x\to\infty$ for each α but $\sup\{|f(x+\alpha)-f(x)|:\alpha\in[0,1]\}\not\to 0$ as $x\to\infty$?

Classification:
26A12 Rate of growth of functions of one real variable