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Asymptotic distribution of normalized arithmetical functions. (In English)
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Let $f(n)$ be an arbitrary arithmetical function and let A_N and $B_N > 0$ be sequences of real numbers such that $B_N \to +\infty$ with N. Let $N\nu_N(n : \dots)$ denote the number of those integers $n \le N$ for which the property stated in the dotted space holds. Our aim in the present paper is to determine sequences A_N and B_N for which

$$\nu_N(n : f(n) - A_N < xB_N) = F(x) + o(1)$$

for all continuity points of a distribution function $F(x)$. We use the method proposed by the second named author [Proc. Amer. math. Soc. 39, 19-25 (1973; Zbl 246.10039)]. That is, we construct additive functions $G_N(n)$ which are close in quadratic mean to $f(n)$ and for which a relation of the form $\nu_N(n : G_N(n) - A_N^* < xB_N) = F(x) + o(1)$ is known to hold. From A_N^* and $G_N(n)$ we then determine A_N. A general theorem of the above nature is proved in the first part while in the second one we discuss in more detail the case when $f(n) = \sum_{d|n} g(d)$ with a given arithmetical function $g(d)$. Notice that if $g(d) = 0$ for all d except when d is the power of a prime number then $f(n)$ is additive while if $g(d)$ is multiplicative, then so is $f(n)$. The most natural extension of the distribution theory of additive and multiplicative functions would therefore be for $f(n)$ of the form given above (perhaps with some restrictions on $g(d)$). We obtain a sufficient condition for the existence of a limit law $F(x)$ for $(f(n) - A_N)/B_N$ with specific A_N and B_N. An example is given for illustration of the method and of the result.

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Classification:
11K65 Arithmetic functions (probabilistic number theory)