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Euclidean Ramsey theorems. I. (In English)

J. Comb. Theory, Ser. A 14, 341-363 (1973).

The abstract states: "The general Ramsey problem can be described as follows: Let A and B be two sets, and R a subset of $A \times B$. For $a \in A$ denote by $R(a)$ the set $\{b \in B \mid (a, b) \in R\}$. R is called r-Ramsey if for any r-part partition of B there is some $a \in A$ with $R(a)$ is one part. We investigate questions of whether or not certain R are r-Ramsey where B is a Euclidean space and R is defined geometrically." - – Let K be a set of k points in Euclidean m-space E^m. Let $R(Kn, n, r)$ denote the property: For any r-coloring of E^n there is a monochromatic set K' congruent to K. (More generally, K' is the image of K under some element of a group H of transformations on E^n.) For example, (the authors prove that) if P is a pair of points distance d apart then $R(P, 2, 7)$ is false [see *L. Moser* and *W.Moser*, Can. Math. Bull. 4, 187-189 (1961)] while $R(P, 2, 3)$ is true. [See *H. Hadwiger* and *H. Debrunner*, Combinatorial Geometry in the Plane (German original 1960; Zbl 089.17302; English transl. with *Klee*, 1964)]. If S_3 is an equilateral triangle of side 1 then $R(S_3, 3, 2)$ is true; if C_2 is a unit square, then $R(C_2, 6, 2)$ is true; if T is any set of three points, then $R(T, 3, 2)$ is true; if T is a 30°–60° right triangle then $R(T, 2, 2)$ is true; if $L = \{(-1, 0), (0, 0), (1, 1)\}$ then $R(L, 3, 2)$ is true, if L_k denotes the configuration of k collinear points separated by unit distance, then $R(L_3, n, 4)$, $R(L_4, n, 3)$, $R(L_5, n, 2)$ are false for all n. — A configuration (set) K is said to be Ramsey if for each r there is an n for which $R(K, n, r)$ is true. K is spherical in E_m if it is embeddable in the surface of a (hyper)sphere. Theorem. If K is not spherical then K is not Ramsey. The proof depends on the lemma: Let $c_1, c_2, \dots, c_k, b$ be real numbers, $b \neq 0$. Then there exists an integer r, and some r- coloring of the real numbers, such that the equation $\sum_{i=1}^k c_i(x_i - x_0) = b$ has no solution $x_0, x_1, \dots, x_k$ where all the x_i have the same color. This lemma extends the fundamental work of *R. Rado* [Proc. London Math. Soc. 2nd Ser. 48, 122-160 (1943)]. Also proved is: if Q (the rationals) is colored with k colors then the equation $(x_1 - y_1)(x_2 - y_2) = 1$ always has solutions with color x_i = color y_i, $i = 1, 2$. The set of vertices of a rectangular parallelepiped in E^n is called a brick. "Every brick is Ramsey" is a corollary of: If $K_1(\subseteq E^n)$ and $K_2(\subseteq E^m)$ are both Ramsey then so is $K_1 \times K_2(\subseteq E^{n+m})$. — The paper combines the best features of exposition, survey, research, and questions for further investigation. It will be read with pleasure by combinatorists, geometers, researchers and students.

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Classification:
05A05 Combinatorial choice problems
05A17 Partitions of integres (combinatorics)
05B30 Other designs, configurations
05-02 Research monographs (combinatorics)