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On the number of times an integer occurs as a binomial coefficient. (In English)

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Let $N(t)$ denote the number of times the integer $t < 1$ occurs as a binomial coefficient; that is, $N(t)$ is the number of solutions of $t = \binom{n}{r}$ in integers n and r. In this note we obtain some additional information about the behavior of $N(t)$. In Theorem 1 we prove that the average and normal order of $N(t)$ is 2; in fact, we prove somewhat more than this, namely, the number of integers t, $1 < t \leq x$, for which $N(t) > 2$ is $0(\sqrt{x})$. [see *G. H. Hardy* and *E. M. Wright*, "Introduction to the theory of numbers" (1960; Zbl 086.25803), p. 263 and p. 356, for the definitions of averageand normal order.] In Theorem 2 we give an upper bound for $N(t)$ in terms of the number $N(t)$ of distinct prime factors of $t : N(t) < 2w(t)\log t/(\log t - w(t)\log\log t)$. Our main result is Theorem 3, in which we show that $N(t) = 0(\log t/\log\log t)$. Finally, in Theorem 4, we consider the related problem of determining the number of representations of an integer as a product of consecutive integers.

Classification:
05A10 Combinatorial functions
11A41 Elemementary prime number theory
05A15 Combinatorial enumeration problems
11B39 Special numbers, etc.
11M99 Analytic theory of zeta and L-functions