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Erdős, Paul; Graham, Ronald L.; Montgomery, P.; Rothschild, B.L.; Spencer, Joel; Straus, E.G.

Euclidean Ramsey theorems. II, III. (In English)

Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 529-558, 559-583 (1975).

[For the entire collection see Zbl 293.00009.]

Continuing the investigations begun in Part I [J. combinat. Theory, Ser. A 14, 341-363 (1973; Zbl 276.05001)], the authors consider several generalizations. In Section 2 of Part II the statement $R(K, n, r)$ defined in Part I (vide Zbl review) is generalized to $R_H(K_1, \dots, K_r, n, r)$: For any r-colouring of E^n there is some i and some K'_i consisting only of points of the i-th color, such that K'_i is the image of K_i under some element of H." Several explicit results are further generalized in the following section, where the question of the existence of many copies of a particular configuration is considered. Section 4 is mainly concerned with the existence of infinite configurations in finite- and infinite-dimensional Euclidean spaces and in real Hilbert spaces. The final section of Part II considers colourings of the edges of Euclidean spaces — i.e. of the unordered point-pairs. For example, it is proved that "Theorem 24. The edges of E^2 can be line colored with 2 colors so that no triangle with all angles at most 90° has all three edges the same color. On the other hand, for every line coloring of E^2 with 2 colors and every $\epsilon > 0$ some triangle with all angles less than $90° + \epsilon$ has all three edges the same color. For very $\epsilon > 0$ and every 2-coloring of the edges of E^2, some triangle with all angles at most $180° + \epsilon$ has all three edges the same color." Part III is devoted to many variations of the special case of $R(\{a, b, c\}, 2, 2)$, and related problems.

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Classification:
05A05 Combinatorial choice problems
05C15 Chromatic theory of graphs and maps
04A20 Combinatorial set theory