Zbl 316.05110
Burr, Stefan A.; Erdős, Paul
On the magnitude of generalized Ramsey numbers for graphs. (In English)
Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 215-240 (1975).

[For the entire collection see Zbl 293.00009.]

Let G and H be graphs and let $r(G,H)$ be the least number so that any edge 2-coloring of K_p (the complete graph on p vertices) with $p \geq r(G,H)$ contains either a subgraph isomorphic with G all of whose edges are colored with the first color or a subgraph isomorphic with H all of whose edges are colored with the second color. We let $r(G)$ denote $r(G,G)$. This paper is devoted to a study of the asymptotic properties of the functions $r(G,H)$ and $r(G)$. Central to this discussion is the concept of an L-set: A set $\{G_1, G_2, \dots\}$ of graphs is called an L-set if there is a constant c so that $r(G_i) \leq c \cdot p(G_i)$, for all i, where $p(G_i)$ denotes the number of vertices of G_i. Also a set of ordered pairs $\{(G_1,H_1),(G_2,H_2),\dots\}$ of graphs is called an L-set if there is a constant c so that $r(G_i,H_i) \leq c \cdot (p(G_i)+p(H_i))$, for all i. Many special cases of, and results related to, the following conjecture are proved in this paper. Conjecture: Any set of graphs or pairs of graphs having bounded arboricity is an L-set. For example Theorem 3.1. Suppose $\{G_1, G_2, \dots\}$ is an L-set having bounded arboricity. Then $\{G_1+K_1, G_2+K_1, \dots\}$ is an L-set. Theorem 3.5. If $n \geq r(K_k)$, $k \geq 2$ and G denotes the graph $K_k \cup (n-k)K_1$, then for some absolute constant c, $k_n + 1 \leq r(G+K_1) \leq kn + cn/k$. Theorem 4.1. There exist graphs $\{G_1, G_2, \dots\}$ and $\{H_1, H_2, \dots\}$ such that $\{(G_1,H_1)(G_2,H_2),\dots\}$ is an L-set but $\{G_1, G_2, \dots\}$ and $\{H_1, H_2, \dots\}$ are no L-sets.

J.E.Graver

Classification:
05C15 Chromatic theory of graphs and maps
05C35 Extremal problems (graph theory)