Zbl 332.10028
Erdős, Paul; Stewart, C.L.
On the greatest and least prime factors of nÜ+1. (In English)
J. London Math. Soc., II. Ser. 13, 513-519 (1976).

The subject of largest prime factors of special sequences of positive integers offers many interessting and challenging problems which are very simple to state. The introduction to the paper gives a short account of these. in the introduction $P(f(1)\dots f(x)) < Cx\log x$ is an over sight; the inequality should be in the opposite direction. The authors prove (denoting by $P(m)$ the largest prime factor of m): Theorem: (i) For all positive integers n,

$$P(n!+1) > n + (1-o(1)\log n/\log\log n.$$

(ii) Let $\epsilon(n)$ be any positive function of n which tends to zero as n tends to infinity. Then for almost all integers n, $P(n!+1) > n+\epsilon(n)n^{1/2}$. (iii) $\limsup_{n\to\infty} P(n!+1)/n > 2+\delta$ where δ is an effectively computable positive constant. The authors also prove: Theorem. Let p_n denote the n-th prime number. Then for infinitely many integers $n(>0)$, $P(p_1\dots p_n+1) > p_{n+k}$ where $k > c\log n/\log\log n$ for some positive absolute constant c. In proving the latter theorem the authors also establish: Theorem. The equations $\prod_{p\le n} p = x^m - y^m$ and $\prod_{p\le n} p = x^m + y^m$ have no solutions in positive integers $x, y, n(> 2)$ and $m(> 1)$.

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Classification:
11N05 Distribution of primes
11N37 Asymptotic results on arithmetic functions
11A41 Elemementary prime number theory
11D41 Higher degree diophantine equations