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Burr, Stefan A.; Erdős, Paul
Extremal Ramsey theory for graphs. (In English)
Utilitas Math. 9, 247-258 (1976).

For graphs G and H the Ramsey number $r(G,H)$ is the least integer p such that in any labelled partition of the complete graph on p vertices into two graphs on the p vertices having disjoint edge sets, either the first contains a copy of G or the second contains a copy of H. The number $r(G,G)$ is denoted by $r(G)$ and called a diagonal Ramsey number. If $\mathfrak{G}$ and $\mathfrak{H}$ are sets of graphs, the authors define $\operatorname{exr}(\mathfrak{G}) = \min_{G\in\mathfrak{G}} r(G)$ and $\operatorname{exr}(\mathfrak{G},\mathfrak{H}) = \min_{G\in\mathfrak{G},H\in\mathfrak{H}} r(G,H)$. C_n, G_n, K_n respectively denote the following classes of graphs: connected on n vertices, n-vertex graphs without isolated vertices, and n-chromatic graphs. $B_{k,l}$ is the set of connected bipartite graphs with maximal independent sets having k and l vertices. In the theorems of §2 $\operatorname{exr}(C_m, K_n)$ and $\operatorname{exr}(G_m, K_n)$ are determined. §3 is concerned with connected graphs with specified chromatic numbers. In the next section the values of $\operatorname{exr}(B_{k,l})$ and $\operatorname{exr}(B_{k,l}, B_{k,l})$ are determined, also those of $\operatorname{exr}(C_n)$ and $\operatorname{exr}(C_n, C_n)$. §5 is concerned with inequalities for $\operatorname{exr}(G_n)$ and $\operatorname{exr}(G_n, G_n)$. The paper closes with several open problems and conjectures.

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Classification:
05C35 Extremal problems (graph theory)
05C15 Chromatic theory of graphs and maps