Zbl 336.10007
Bleicher, Michael N.; Erdős, Paul
Denominators of Egyptian fractions. II. (In English)
Ill. J. Math. 20, 598-613 (1976). [0019-2082]

[Part I, cf. J. Number Theory 8, 157-168 (1976; Zbl 328.10010).] A positive fraction a/N is said to be written in Egyptian form if we write $a/N = 1/n_1 + 1/n_2 + \ldots + 1/n_k$, $0 < n_1 < n_2 < \ldots < n_k$, where the n_i are integers. Among the many expansions for each fraction a/N there is some expansion for which n_k is minimal. Let $D(a, N)$ denote the minimal value of n_k. Define $D(N)$ by $D(N) = \max\{D(a, N) : 0 < a < N\}$. We are interested in the behavior of $D(N)$. In this paper we show that on the one hand for a prime P large enough that $\log_{2r} P \geq 1$,

$$D(P) \geq \frac{P \log P \log_2 P}{\log_{r+1} P \sum_{j=4}^{r+1} \log_j P}$$

and on the other hand that for $\epsilon > 0$ and N sufficiently large (Theorem 1 and its corollary yield more precise statements),

$$D(N) \leq (1 + \epsilon) N (\log N)^2.$$

We conjecture that the exponent 2 can be replaced by $(1 + \delta)$ for $\delta > 0$. As part of the proof of the above results we need to analyze the number of distinct subsums of the series $\sum_{i=1}^{N} 1/i$, say $S(N)$. We show that whenever $\log_{2r} N \geq 1$,

$$\frac{\alpha N}{\log N} \prod_{j=3}^{r} \log_j N \leq \log S(N) \leq \frac{N \log_r N}{\log N} \prod_{j=3}^{r} \log_j N$$

for some $\alpha \geq 1/e$.

Classification:
11A63 Radix representation
11D85 Representation problems of integers
11D61 Exponential diophantine equations