
Zbl 352.10027
Diamond, Harold G.; Erdős, Paul
A measure of the nonmonotonicity of the Euler phi function. (In English)
Pac. J. Math. 77, 83-101 (1978). [0030-8730]

Let f be a real valued arithmetic function satisfying $\lim_{n\to\infty} f(n) = +\infty$. Define another arithmetic functions $F = F_f$ by setting

$$F_f(n) = \#\{j < n : f(j) \geq f(n)\} + \#\{j > n : f(j) \leq f(n)\}.$$

The size of the values assumed by the function F provides a measure of the nonmonotonicity of f. In particular, F is identically zero if an only if f is strictly increasing. In the present article $f = \varphi$, Euler's functions and F_φ is written as F. It is shown that $F(n)/n$ is asymptotically represented as $h(\varphi(n)/n)$, where h is a certain convex function. Using this representation it is shown that $F(n)/n$ has a distribution function. The functions $\max_{n\leq x} F(n)$ and $\min_{n>x} F(n)$ are studied and conditions on $\varphi(n)/n$ are found which lead to large and small values of $F(n)/n$.

Classification:
11K65 Arithmetic functions (probabilistic number theory)
11N37 Asymptotic results on arithmetic functions
11A25 Arithmetic functions, etc.