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On changes of signs in infinite series. (In English)

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The main theorem of this paper is the following: Theorem2: Let $\{a_n\}$ be a sequence of positive real numbers monotonically decreasing to 0 such that $\Sigma a_n = \infty$. Let s_{nj}, $n = 1, 2, \dots, j = 0, \dots, n! - 1$ be real numbers such that

$$\sum_{j=0,\ J\equiv d(\mod (n-1)!)}^{n!-1} s_{nj} = s_{n-1,d'} \quad n = 2, 3, \dots, \quad 0 \le d \le (n-1)! - 1.$$

Then there exists signs $\varepsilon(n) = \pm 1$, $n = 1, 2, \dots$ such that

$$\sum_{k=1,\ k\equiv j(\mod n!)}^{\infty} \varepsilon(k) a_k = s_{nj}$$

for $n = 1, 2, \dots$ and $0 \le j \le n! - 1$. Under the same assumptions on $\{a_n\}$, a consequence (Theorem 1) of the above theorem is that there exist signs $\varepsilon(n) = \pm 1, n = 1, 2, \dots$ such that for every integer $m \ge 1$ and every integer $0 \le v \le m - 1$,

$$\sum_{n\equiv b(\mod m)} \varepsilon(n) a_n = 0.$$

This deduction shows that the result:

$$\sum_{n=1}^{\infty} |a_n| < \infty, \quad A_m \equiv \sum_{n\equiv 0(\mod m)} a_n = 0$$

for all $m = 1, 2, \dots \Rightarrow a_1 = a_2 = \dots = 0$, is sharp when $\{|a_n|\}$ is monotonic. An interesting consequence of the main theorem its that there is a non-trivial power series $\Sigma a_n z^n$ which vanishes for every $z = e^{2\pi i\theta}$, θ rational. Five interesting problems are also posed by the authors.

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11B83 Special sequences of integers and polynomials
40A05 Convergence of series and sequences
11B39 Special numbers, etc.

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changes of signs in infinite series; sequence of positive real numbers