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Erdős, Paul; Szabados, J.
On the integral of the Lebesgue function of interpolation. (In English)
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Let $-1 \leq X_1 < X_2 < \cdots < X_n \leq 1$ be n distinct numbers in $(-1, +1)$. $\omega(x) = \prod_{i=1}^{n}(X - X_i)$, put

$$\ell_k(X) = \frac{\omega(X)}{\omega'(X_k](X - X_k)}.$$

$\ell_k(X)$ are the fundamental functions of Lagrange interpolation polynomials, $\ell_k(X_k) = 1$ and $\ell_k(X_1) = 0$ for $i \neq k$. The author pove

$$\sum_{k=1}^{n} \int_{-1}^{+1} |\ell_k(X)| d(X) > c \log n \tag{1}$$

for a certain absolute constant $c > 0$. The proof is not very simple and the best value of c is not determined. It seems a reasonable guess that asymptotically (1) is a minimum if the X_i are the roots of the Chebyshev polynomial $T_n(X)$. But we have not been able to prove this.

Classification:
41A05 Interpolation

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Lebesgue function; integration of interpolation functions; Lagrange interpolation polynomials