**Zbl 393.10047**
**Erdős, Paul; Hall, R.R.**
*On some unconventional problems on the divisors of integers.* (In English)
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The authors prove several theorems concerning the divisors of an integer. Let $\tau(n)$ be the number of divisors of $n$, and let $d_1,\dots,d_\tau$ be all divisors of $n$, ordered so that $1=d_1<d_<\cdots<d_\tau=n$. Let $f(n)=\text{card}\{i:(d_i,d_{i+1})=1\}$. Next, let $\tau_k(n)$ be the number of divisors of $n$ of the form $d=t(t+1)\dots(t+k-1)$, and let $t_k(n)=\min\{t\geq 1: n|t(t+1)\dots(t+k-1)\}$. The following results are proven:
Theorem 1. For every $\varepsilon>0$ and $x>x_0(\varepsilon)$

$$\max f(m)>\exp((\log\log x)^{2-\varepsilon}).$$

Theorem 2. for each $k\geq 2$ and every fixed $A<e^{1/k}$ we have $\tau_k(n)>(\log n)^A$ infinitely often.
Theorem 3.

$$\frac{1}{x}\sum_{n\leq x}t_2(n)\ll x\frac{\log\log\log x}{\log\log x}$$

The last proven result involves the divisors of two integers. We say that two integers $m$ and $n$ interlock if every pair of divisors of $n$ are separated by a divisor of $m$, and conversely (except for 1 and the smallest prime factor ofmn). An integer $n$ is said to be separable if there exists an integer $m$ such that $m$ and $n$ interlock, and let $A(x)$ be the number of separable $n\leq x$. It is proven:
Theorem 4. For every fixed $c'>0$ and sufficiently large $x$ we have

$$A(x)>c'x/\log\log x.$$

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