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Some number theoretic problems on binomial coefficients. (In English)

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In this paper some problems which are simple to state but probably difficult to solve are posed concerning binomial coefficients. Let $P(m,n)$ denote the greatest prime factor of (m,n). Then the authors conjecture that if $1 \le j \le n/2$ then $P(\binom{n}{i},\binom{n}{j}] \ge i$ with equality holding only in a few special cases (several of which are given). If $f(n)=\min_{1<j\le n/2}(n,[\binom{n}{j}]))$ it is not difficult to show that $f(n)\ge p(n)$ is the smallest prime factor of n, and that if n is not a prime power then $f(n)\le n/P(n)$ where $P(n)$ is the greatest prime power which divides n. The authors remark that it would be of interest to characterize those n for which $f(n)=n/P(n)$. (For example, $f(30)=6$.) They also mention that it seems likely that $f(n)>\sqrt{n}$ for infinitely many n.

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11A05 Multiplicative structure of the integers
05A10 Combinatorial functions
11A41 Elemementary prime number theory
00A07 Problem books

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