Zbl 433.04002

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Some remarks on subgroups of real numbers. (In English)

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Stanisław Hartman asked: Is there a group G of real numbers which is of measure 0and of second category? (Problem of Stanisław Hartman). In the present note the author gives an affirmative answer to this problem assuming the continuum hypothesis: it is sufficient to consider as an ω_1-sequence A_n $(n < \omega_1)$ the system of all subsets of R which are F_sigma and of first category and then to construct a strictly increasing ω_1-sequence G_n of countable subgroups of $(R, +)$ such that $G_n \cap \cup_{i<n} G_i = G_r \cap \cup_{j<n} A_j$ for $n < r < \omega_1$; then $G := \cup G_n (n < \omega_1)$ is a requested group. Dually, the author establishes (under CH) the existence of a subgroup of R that is of the first category but not of measure 0. In this connexion let us recall a remarkable result of *W.Sierpiński* [Fundam. Math. 22, 276-280 (1934; Zbl 009.20405), and also pp. 207-210 in his Oeuvres choisies. Tome III (1976; Zbl 316.01012)]. There is a permutation $p \in R!$ which is a $1-1$ mapping between the system of all sets $\subset R$ of measure zero and the system of all subsets of R which are of the first category. *P.Erdős* [Ann. Math. II. Ser. 44, 643-646 (1943; Zbl 060.13112)] improved this result of Sierpiński answering in affirmative a question of Sierpiński whether moreover p could satisfy $p^{-1} = p$. The author states that a similar result holds if one requires that G be a field; in this case the move group $\to$ ring, field is easy; is such a move possible in the following statement? EV: For every $0 \leq \alpha \leq 1$ there is a group of real numbers of Hausdorff dimension α [cf. *P.Erdős* and *B.Volkmann*, J. Reine Angew. Math. 221, 203-208 (1966; Zbl 135.10202)].

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04A15 Descriptive set theory
04A30 Continuum hypothesis and generalizations
28A05 Classes of sets
54F45 Dimension theory (general topology)

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