Zbl 434.05001

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Combinatorial problems on subsets and their intersections. (In English)

Studies in foundations and combinatorics, Adv. Math., Suppl. Stud., Vol. 1, 259-265 (1978).

[For the entire collection see Zbl 432.00006.]

Let $|S| = n$, $m(n, l_1, l_2, k)$ (respectively, $m'(n, l_1, l_2, k)$) denote the cardinality of the largest family of subsets $A_i \subset S$ satisfying $|A_i| = k$ (respectively, $|A_i| \le k$) and $|A_{i_1} \cap A_{i_2}| = l_1$ or l_2. In this paper we prove
(a) $m(n, 0, l_2, k) \le \binom{n}{2}$, $m'(n, 0, l_2, k) \le \binom{n}{2} + n + 1$; equality if $k = 2$;
(b) $m(n, 0, l_2, k) \le n$, if $l_2 \nmid k$, with equality for an infinity of n.
For $n \ge n_0(k)$ we show that
(a) $m(n, l_1, l_2, k) \le \binom{n-l_1}{2}$, $m'(n, l_1, l_2, k) \le \binom{n-l_1}{2} + (n - l_1) + 1$;
(b) more exactly, $m(n, l_1, l_2, k) \le \left[\frac{n-l_1}{k-l_1}\left[\frac{n-l_2}{k-l_2}\right]\right]$, with equality for an infinity of n.

Classification:
05A05 Combinatorial choice problems

Keywords:
family of subsets