Zbl 466.10037

Erdős, Paul; Nicolas, Jean-Louis

Sur la fonction: Nombre de facteurs premiers de N.

On the function: Number of prime factors of N. (In French)

Enseign. Math., II. Ser. 27, 3-27 (1981). [0013-8584]

This paper considers several problems concerning the functions $\omega(n)$ and $\Omega(n)$. The following are proved: 1) Let $Q_1(x)$ be the number of $n \le x$ such that $\omega(n) \le \omega(m)$ whenever $m \le n$. Then $(\log x)^{1/2} \ll \log Q_1(x) \ll (\log x)^{1/1}$. 2) For any fixed $c > 0$ has

$$\#\left\{n \le x; \omega(n) > \frac{c \log x}{\log\log x}\right\} x^{1-c+O(1)}.$$

3) $\lim\sup(\log n)^{-1}(\Omega(n)+\Omega(n+1)) = (\log 2)^{-1}$. 4) There exist infinitely many n for which $m - \omega(m) < n - \omega(n)$ whenever $m < n$ and $m - \omega(m) > n - \omega(n)$ whenever $m > n$. 5) If $\alpha > 1$ is constant there is an asymptotic formula for $\#\{n \le x; \omega(n) > \alpha \log\log x\}$, correct to within a factor $1 - O((\log\log x)^{-1})$. The methods used are largely elementary, but an ineffective result on Diophantine approximation is also needed.

D.R.Heath-Brown

Classification:
11N37 Asymptotic results on arithmetic functions
11N05 Distribution of primes

Keywords:
number of prime factors; largely composite; total number of prime factors; asymptotic formula