Zbl 504.05052
Erdős, Paul; Fajtlowicz, Siemion
On the conjecture of Hajos. (In English)
Combinatorica 1, 141-143 (1981). [0209-9683]

G.Hajós [Wiss. Z. Martin-Luther-Univ. Halle-Wittenberg, Math.- Naturw. Reihe 10, 113-117 (1961; Zbl 094.17602), pp. 116-117] conjectured that every s-chromatic graph contains a subdivision of K_s, the complete graph on s vertices. This conjecture was disproved ins a paper by *P.A.Catlin* [J. Comb. Theory, Ser. B 26, 268-274 (1979; Zbl 385.05033)]. In the present paper it is shown by probabilistic methods that the Hajós conjecture fails for almost all graphs. More precisely, let $G = G(n)$ be a graph of n vertices. Denote by $\chi(G)$ the chromatic number of G and by $\sigma(G)$ the largest integer ℓ such that G contains a subdivision of K_ℓ. Put $H(G) = \chi(G)/\sigma(G)$ and $H(n) = \max_{G(n)} H(G(n))$ (hence, the Hajós conjecture says $H(n) = 1$). In the present paper it is shown that there exists an absolute constant c such that $H(n) > C\sqrt{n}/\log n$ holds for almost all labelled graphs with n vertices.

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05C15 Chromatic theory of graphs and maps
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