Zbl 513.10043

Canfield, E.R.; Erdős, Paul; Pomerance, Carl

On a problem of Oppenheim concerning "Factorisatio Numerorum". (In English)

J. Number Theory 17, 1-28 (1983). [0022-314X]

Denote by $f(n)$ the number of factorizations of a positive integer n into factors exceeding 1, the order of the factors being immaterial. In this interesting paper, the authors establish a good estimate for the maximal order of $f(n)$, thus correcting a result of *A.Oppenheim* [J. Lond. Math. Soc. 1, 205-211 (1926); ibid. 2, 123-130 (1927)]; their estimate is of the form

$$n \exp(-\log n (\log n)^{-1} \log_3 n(1 + E(n)))$$

where $E(n) = o(1)$ as $n \to \infty$ and is given rather more explicitly in the paper, and where $\log_k n$ denotes the k-fold iterated logarithm. A new lower bound for $\Psi(x, x^{1/u})$, the number of positive integers $n \le x$ with no prime factor exceeding $x^{1/u}$, is also derived (and applied), namely

$$\Psi(x, x^{1/u}) \ge x \exp(-u(\log u + (\log_2 u - 1)\left(1 + \frac{1}{\log u}\right) + ..elke..(\log_2^2 u \log^{-2} u)))$$

for $x \ge 1$, $u \ge 3$. The paper concludes with an investigation of the largest prime divisors of highly factorable numbers n, i.e. those n for which $f(m) < f(n)$ whenever $m < n$ (in which case $f(n)$ has maximal order). The 118 highly factorable numbers up to 10^9 are listed, and the algorithm used to obtain them described. Some additional questions are raised.

E.J.Scourfield

Classification:
11N37 Asymptotic results on arithmetic functions
11N05 Distribution of primes
11B83 Special sequences of integers and polynomials
11A25 Arithmetic functions, etc.

Keywords:
number of factorizations of positive integer; maximal order; lower bound for Psi-function; largest prime divisors of highly factorable numbers