Zbl 523.10023
Erdős, Paul; Sarkoezy, A.
On almost divisibility properties of sequences of integers. I. (In English)
Acta Math. Hung. 41, 309-324 (1983). [0236-5294]

Let $|\alpha|$ denote the distance from the real number α to the nearest integer. we say, the positive real number b is ε- divisible by the positive real number a (and we then write $a|_\varepsilon b$), if $|b/a| < \varepsilon$. The following theorem will be proved. Let $\varepsilon > 0$, $n > n_1(\varepsilon)$ a positive integer, t a real number such that $n < t \leq \exp(\log^{5/4} n/\log\log n)$. Let further

$$k = \begin{cases} [2\log t/\log n] - 3 & \text{if } 2 \leq \log t/\log n < c_1, \\ [\log t/\log n + 0.5] & \text{if } \log t/\log n \geq c_1, \end{cases}$$

where c_1 is a certain positive absolute constant. With

$$F = \begin{cases} n & \text{if } n < t < n^2 \\ [n^{1-1/2^{k+2}}] & \text{if } n^2 \leq t < n^{c_1} \\ [(n^{k+5/2}/t)^{1/(k+2)}] & \text{if } t \geq n^{c_1} \end{cases}$$

there exists a positive integer j such that $1 \leq j \leq P$ and $(n+j)|_\varepsilon t$.
The main tool are estimates of certain trigonometric sums. On the other hand it will be shown, that for $0 < \varepsilon < 1/4$, $\delta > 0$ and $n > n_2(\varepsilon)$ there exists a real number t such that $n < t < \exp((2+\delta)n)$ and there does not exist an integer j satisfying $l \leq j \leq n$ and $(n+j)|_\varepsilon t$. This leads to the inequality

$$\exp(\log^{5/4} n/\log\log n) \leq f(n,\varepsilon) \leq \exp((2+\delta)n) \quad (n \geq n_0(\varepsilon))$$

where $f(n,\varepsilon)$ denotes the infimum of the real numbers $t > n$ such that for any integer $l \leq j \leq n$ t is not ε-divisible by $n+j$.

D.Leitmann

Classification:
11L03 Trigonometric and exponential sums, general
11A05 Multiplicative structure of the integers

Keywords:
almost divisibility; sequences of integers; epsilon-divisibility; estimates of trigonometric sums