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Erdős, Paul; Tenenbaum, G.
Sur les diviseurs consecutifs d'un entier.
On consecutive divisors of an integer. (In French)
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Let $1 = d_1 < d_2 < \cdots < d_{\tau(n)} = n$ denotes the divisors of n. The well known conjecture of the first author that almost all positive integers n have a pair d, d' of divisors such that $d < d' \leq 2d$ has prompted various investigations into the behaviour of pairs of divisors of an integer.
In this interesting paper; the authors consider some properties of pairs of consecutive divisors d_i, d_{i+1} of n. If θ is a real bounded function defined on $(0,1)$ and if $F(n;\theta) = \sum_{1\leq i<\tau(n)} \theta(d_1/d_{i+1})$, it is established in Theorem 1 that $F(n;\theta)/\tau(n)$ has a distribution function. An asymptotic formula for $\sum n \leq xF(n;\theta)$ is derived in Theorem 2 for a class of functions θ and in Theorem 3 for the function given by $\theta(t) = t^r$, the result here being uniform for $r \log x \gg 1$. The authors also study the sums $\sum_{n\leq x} f(n)$, $\sum_{n\leq x} g(n)$ where $f(n)$, $g(n)$ denote the number of pairs d_i, d_{i+1} of divisors of n with the property that $(d_i, d_{i+1}) = 1$, $d_i|d_{i+1}$, respectively. By choosing the function θ appropriately, it follows from Theorem 1 that $g(n)/\tau(n)$ has a distribution function, as conjectured by the authors in [Ann. Inst. Fourier 31, No. 1, 17-37 (1981; Zbl 437.10020)]. The proofs in this paper depend on some rather intricate handling of various double sums involving the characteristic functions associated with certain divisibility properties, and are rather complicated.

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Classification:
11N37 Asymptotic results on arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)

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arithmetical function; pairs of consecutive divisors; asymptotic formula; distribution function; number of pairs of divisors