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Congruent subsets of infinite sets of natural numbers. (In English)

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If A is an infinite subset of the set $\mathbb{N}$ of natural numbers, $A(x)$ denotes the number of elements of A which are $\leq x$. The main theorem states: If k and n are given natural numbers > 1 and if $A(x) \geq \epsilon \cdot x^{1-1/n}$ for some positive ϵ and all x of a final segment of $\mathbb{N}$, then there exist k disjoint n-element subsets of A which are congruent by translation. Of course, this also implies that n disjoint k-element subsets of A exist which are congruent by translation. This improves an earlier result of the first author for $k = n = 2$ on B_2- sequences, which was published in a paper of *A.Stöhr* [J. Reine Angew. Math. 194, 111-140 (1955; Zbl 066.03101)].

One obtains the corollary that for every two natural numbers k, n the set of prime numbers has k disjoint n-element subsets which are congruent by translation. Concerning the sharpness of the theorem there holds: If $0 < \alpha < 1 - 1/k - 1/n + 1/kn$ then for all sufficiently large natural numbers m there exists a subset of $\{1, ..., m\}$ which has at least m^α elements but no k disjoint congruent n-element subsets.

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