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On the representing number of intersecting families. (In English)

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The paper presents a generalization of well-known results: Theorems of Erdős-Ko-Rado (1961) and Hilton-Milner (1967). Let $\mathcal{M}$ be a family of sets, and R a single set. R is said to represent $\mathcal{M}$ or be a representing set for $\mathcal{M}$ if $R \cap X \neq 0$ for all $X \in \mathcal{M}$ has representing number r if r is the cardinality of a smallest set representing. Let $\begin{pmatrix} M \\ k \end{pmatrix}$ denote the collection of all k-subsets of a finite set. A family is intersecting if any two members of it have a non-trivial intersection. Theorem. Denote by $g(n; r, k)$ the maximal cardinality of an intersecting family $\mathcal{M} \subseteq \begin{pmatrix} M \\ k \end{pmatrix}$ of an n-set with representing number r, $1 \leq r \leq k \leq n$. Then there are constants $c_{r,k}$, $C_{r,k}$ only depending on r and k, such that $c_{r,k}n^{k-r} \leq g(n; r, k) \leq C_{r,k}n^{k-r}$. The precise value of $g(n; i, k) = \binom{n-1}{k-1}$ and $g(n; 2, k) = \binom{n-1}{k-1} - \binom{n-k-1}{k-1}$ are given by Theorems Erdős-Ko-Rado and Hilton-Milner, correspondingly. Some estimations of $g(k) = g(n; k, k)$, which for $n \geq n_0(k)$ is independent of n, are obtained. The authors put the following questions: first, improve the bounds on $g(k)$, second, estimate the value of $n_0(k)$. See also: *I. Anderson* and *A. J. W. Hilton*, The Erdős-Ko-Rado theorem with valency conditions, Q. J. Math., Oxf. II. Ser. 37, 385-390 (1986; Zbl 619.05037).

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05A05 Combinatorial choice problems
04A20 Combinatorial set theory

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