Zbl 664.10025
Alladi, K.; Erdős, Paul; Vaaler, J.D.
Multiplicative functions and small divisors. II. (In English)
J. Number Theory 31, No.2, 183-190 (1989). [0022-314X]

Let k be an integer ≥ 2 and h a multiplicative function satisfying $0 \leq h(p) \leq 1/(k-1)$ for every prime p. The authors show that, for any squarefree integer h,

$$(*) \quad \sum_{d|n} h(d) \leq (2k + o(1)) \sum_{d|n; d \leq n^{1/k}} h(d),$$

where $o(1)$ is a quantity that tends to zero as the number of prime divisors of n tends to infinity. In part I of the present paper [Analytic number theory and diophantine problems, Prog. Math. 70, 1-13 (1987; Zbl 626.10004)] the authors had obtained a similar result but under the stronger hypothesis that $0 \leq h(p) \leq c$ for some fixed constant $c < 1/(k-1)$.

The proof of (*) rests on a deep theorem of Baranyai on hypergraphs. The authors give heuristic arguments suggesting that (*) remains true with the constant $4 + o(1)$ in place of $2k + o(1)$ and for any real $k \geq 2$.

{Note: A result similar to the authors' had been obtained very recently by *B. Landreau* [C. R. Acad. Sci., Paris, Sér. I 307, No.14, 743- 748 (1988; Zbl 658.10053)].}

A.Hildebrand

Classification:

11N37 Asymptotic results on arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)
11A25 Arithmetic functions, etc.

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divisors; multiplicative function; theorem of Baranyai on hypergraphs