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On the number of partitions of n without a given subsum. I. (In English)

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Let $R(n,a)$ be the number of partitions of $n = n_1 + ... + n_t$, where subsums $n_{i_1} + ... + n_{i_j}$ are all different from a. The authors examine $R(n,a)$ for a depending on n and smaller than $\lambda_0\sqrt{n}$, λ_0 a small positive constant. They prove that there exists $\lambda_0 > 0$ such that uniformly for $1 \le a \le \lambda_0\sqrt{n}$, when n goes to infinity,

$$(1)\quad \log(\frac{R(n,a)}{p(n)}) \le (\psi(a)\log\frac{\pi a}{\sqrt{6n}}) + O(1/\sqrt{n}),$$

$$(2)\quad \log\binom{R(n,a)}{p(n)} \ge \psi(a)\log\frac{\pi a}{\sqrt{6n}} - \gamma_a a + O(a^2/\sqrt{n})$$

where $\psi(a) = \lfloor \frac{a}{2} + 1 \rfloor$, p(n) is the unrestricted partition function and $\gamma_a = 1/2$ if a is odd, and $\gamma_a = 1/2 + \log 3 - (7/6)\log 2 + (c\log a)/a = +0.79... + (c\log a)/a$ if a is even, where c is a fixed constant.

Further, if $Q(n,a)$ is the above function $R(n,a)$ with the restriction that each part occurs at most once, then there exists $\lambda_1 > 0$ such that uniformly for $1 \le a \le \lambda_1\sqrt{n}$, $\log(\frac{Q(n,a)}{q(n)}) \ge -(a/6)\log(16/3) - \log 3 + O(a^2/\sqrt{n})$.

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05A17 Partitions of integres (combinatorics)
05A15 Combinatorial enumeration problems
11P81 Elementary theory of partitions

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number of partitions