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Erdős, Paul; Tenenbaum, G.

Sur les densités de certaines suites d'entiers.

On densities of certain sequences of integers. (In French)

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Some distinguished number theorists have been attracted to systematically studying the distribution and other global and local properties of the divisors of an integer, and amongst those in the forefront are the authors of the paper under review, who have made major contributions to this field. Some of this work is described in [*R. R. Hall* and *G. Tenenbaum*, Divisors (Cambridge Tracts Math. 90) (1988; Zbl 653.10001)]. Denote the set of distinct prime divisors of n by $\{p_j(n) : 1 \le j \le \omega(n)\}$ and the set of distinct positive divisors of n by $\{d_j(n) : 1 \le j \le \tau(n)\}$; in this article, the authors turn their attention to the interesting question of investigating the quantities

$$\lambda_k(p) = \text{dens}\{n : \ p_k(n) = p\}, \qquad \Lambda_k(d) = \text{dens}\{n : \ d_k(n) = d\},$$

and are able to obtain more precise results for the former than the latter. Their first Theorem expresses $\lambda_k(p)$ uniformly in terms of certain analytic functions for $1 \le k \le p^{1-\epsilon}$ ($\epsilon > 0$), and, in Corollaries 4 and 5, they determine explicitly formulae for $\max_k \lambda_k(p)$ as $p \to \infty$, $\max_p \lambda_k(p)$ and for the values at which these maxima are obtained. They go on to establish upper and lower bounds for the corresponding maximal quantities associated with the function $\Lambda_k(d)$.

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Classification:
11N05 Distribution of primes
11N37 Asymptotic results on arithmetic functions
11B83 Special sequences of integers and polynomials
11K65 Arithmetic functions (probabilistic number theory)

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density; local behaviour; prime factors; distinct prime divisors; distinct positive divisors; upper and lower bounds