Zbl 695.10048

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On the number of sets of integers with various properties. (In English)

Number theory, Proc. 1st Conf. Can. Number Theory Assoc., Banff/Alberta (Can.) 1988, 61-79 (1990).

[For the entire collection see Zbl 689.00005.]

In this interesting paper, the authors investigate the general problem of determining the number of subsets of $[1, n]$ which satisfy a given constraint. These are classified under the headings of (i) additive conditions, (ii) multiplicative conditions, (iii) divisibility and common factors and (iv) miscellaneous problems. A brief resumé is:

(i) (a) sum-free sequences ($a_i + a_j \neq a_k$). Here, for example, the authors show that the number of sum-free sequences of $[1, n]$ whose least element is $> n/3$ does not exceed $c2^{n/2}$ for some absolute constant c. (b) Sidon sequences ($a_i + a_j \neq a_k + a_\ell$ for { i,j} $\neq \{k, \ell\}$. (c) All partial sums distinct ($\sum \epsilon_i a_i$ are distinct, $\epsilon_i = 0, 1$).

(ii) (a) Product-free sequences ($a_i a_j \neq a_k$). (b) Pairwise products distinct, and related constraints.

(iii) (a) Requiring divisibility ($a_i | a_j$ for $i < j$). (b) Forbidding divisibility ($a_i \nmid a_j$ for $i \neq j$). (c) Any two terms coprime. (d) No two terms coprime.

(iv) (a) $(a_i + a_j) \nmid a_i a_j$. (b) No k-term arithmetic progression. (c) $\sum 1/a_i \leq s$.

In each of these topics, the authors discuss the best results known, prove theorems of their own or conjecture what might be expected to be true.

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Classification:
11B13 Additive bases
11B25 Arithmetic progressions
11B99 Sequences and sets
11B75 Combinatorial number theory

Keywords:
number of subsets; additive conditions; multiplicative conditions; divisibility; common factors; sum-free sequences; Sidon sequences; Product- free sequences; arithmetic progression