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Erdős, Paul; Sárközy, A.

On a conjecture of Roth and some related problems. II. (In English)

Number theory, Proc. 1st Conf. Can. Number Theory Assoc., Banff/Alberta (Can.) 1988, 125-138 (1990).

[For the entire collection see Zbl 689.00005.]

Let $\mathcal{A}_1, ..., \mathcal{A}_k$ be a partition of the set of natural numbers into disjoint classes, and let $\mathcal{B}$ be the set of integers that have a representation in the form aa' where $a, a' \in \mathcal{A}_i$ for some i. It is proved that for a fixed number M the minimal number of elements (over all partitions) of $\mathcal{B}$ up to M is between $M(\log M)^{-\alpha}$ and $M(\log M)^{-\beta}$ with some constants $0 < \beta < \alpha$, but $\sum_{b \in B} 1/b > (c/k) \log M$ with an absolute constant $c > 0$. The second result implies that for a fixed partition the upper density of $\mathcal{B}$ is $> c/k$, and it is proved that it can be $< c'/k$ with another constant c'. The lower density is also positive, but it is proved that already for $k = 3$ it cannot be estimated from below from a positive constant independent of the concrete partition; for $k = 2$ this problem remains undecided.

The corresponding additive problem was investigated by the authors and *V. T. Sos* in Part I of this paper [Irregularities of partitions, Pap. Meet., Fertod/Hung. 1986, Algorithms. Comb. 8, 47-59 (1989; Zbl 689.10061)].

I.Z.Ruzsa

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