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Domination in colored complete graphs. (In English)

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A 2-colored graph G is a graph with edges colored red or blue. A set $X \subset V(G)$ r-dominates (b-dominates) $Y \subset V(G)$ if $X \cap Y = \emptyset$and for each $y \in Y$ there exists $x \in X$ such that the edge (x, y) is red (blue). The set $X \subset V(G)$ dominates $Y \subset V(G)$ if either X r-dominates Y or X b-dominates Y. A set A on t vertices is said to dominate all but at most k vertices of G if A dominates B and $|V(G)-A-B| \leq k$. The following conjecture is due to the first author and *A. Hajnal* [Ramsey type theorems, Preprint (1987), Discrete Appl. Math. 25, No.1/2, 37-52 (1989; Zbl 715.05052)]. For given positive integers n, t, any 2-colored complete graph K_n of order n has a set X_t of at most t vertices dominating all but at most $n/2^t$ vertices of K_n. The authors prove this conjecture by proving the following more general result. Let $G = [X, Y]$ be a 2- colored complete bipartite graph, t be a nonnegative integer, and $\beta \in (0, 1)$. Then at least one of the following two statements is true:

1. Some subset of at most t vertices of X r-dominates all but at most $\beta^{t+1}(|X|+|Y|)$ vertices of Y.
2. Some subset of at most t vertices of Y b-dominates all but at most $(1-\beta)^{t+1}(|X| + |Y|)$ vertices of X.

The proof of this result is constructive, in fact, it is a greedy-type low- order polynomial algorithm which finds the required (red or blue) dominating set.

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r-domination; dominating set