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On a theorem of Besicovitch: Values of arithmetic functions that divide their arguments. (In English)

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The authors' starting point is the following result of C. N. Cooper and R. E. Kennedy: Given an arithmetic function $f:\mathbb{N}\to\mathbb{N}$, the set of integers n with the property $f(n)|n$ has asymptotic density zero, if $\mu(x)=x^{-1}\sum_{n\le x}f(n)\to\infty$ and $\sigma(x)/\mu(x)\to 0$, where σ (x) denotes the standard deviation.
The authors prove (Theorem 1) the same conclusion in a rather simple way, if f has normal order g, with $g\nearrow\infty$ and $g(n)/n\to 0$.
Next they show (Theorem 2) that, for a function g satisfying the conditions given above, the set of integers n with a divisor in the interval $(g(n),2g(n)]$ has asymptotic density zero.
The following lemma, used in the proof of Theorem 2, seems to be of independent interest: Denote by $\Omega_z(n)$ the number of prime and prime-power factors of n not exceeding z. If $z\ge 3$, and $0<C<D$, then the number of integers n in the interval $(C,D]$ with the property $|\Omega_z(n)-\log\log z|\ge 1/3\cdot\log\log z$ may be uniformly estimated by

$$\ll (D-C)\cdot(\log\log z)^{-1}+z\cdot(\log z\cdot\log\log z)^{-1}.$$

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11N37 Asymptotic results on arithmetic functions
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values of arithmetical functions; integers with divisors in some prescribed intervals; theorem of Cooper and Kennedy; Turán-Kubilius inequality; asymptotic density zero