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On isolated, respectively consecutive large values of arithmetic functions. (In English)

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This paper is divided into two parts. In the first part, the authors investigate the occurrence of isolated large values of the arithmetic functions $\omega(n)$, $\Omega(n)$, $d(n)$, and $\sigma(n)$. Given an arithmetic function $f(n)$ and $x \geq 1$, the authors define $G = G(f, x)$ as the largest integer such that the inequality $f(n) > \sum_{0<|i|\leq G} f(n+i)$ holds for some positive integer $n \leq x$, and obtain estimates for this quantity when f is one of the above functions. For example, in the case $f = \omega$ the authors show that

$$\frac{\log x}{(\log_2 x)^2} \ll G(\omega, x) \ll \frac{\log x}{\log_2 x \log_3 x},$$

where $\log_k x$ denotes the k fold iterated logarithm, and for $f = \sigma$ they derive the asymptotic formula

$$G(\sigma, x) \sim \frac{3e^\gamma}{\pi^2} \log_2 x \qquad (x \to \infty).$$

The second part of the paper is devoted to the study of consecutive large values of the above arithmetic functions. Setting $M(f, x) = \max_{n\leq x} f(n)$, $T(f, x) = \max_{n\leq x}(f(n-1) + f(n))$, the authors show, for example, that

$$T(\Omega, x) \geq M(\Omega, x) + \exp\left\{(\log 2 - \varepsilon)\frac{\log_2 x}{\log_3 x}\right\}$$

holds for any given $\varepsilon > 0$ and arbitrarily large values χ. Surprisingly, as the authors remark, the case of the function $\omega(n)$ appears to be much more difficult, and even the weakest non-trivial estimate of this type, namely $\limsup_{x\to\infty}(T(\omega, x) - M(\omega, x)) = \infty$, remains an open problem.

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Classification:
11N37 Asymptotic results on arithmetic functions
11N64 Characterization of arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)

Keywords:
occurrence of isolated large values; arithmetic functions; asymptotic formula; consecutive large values