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Sets of prime numbers satisfying a divisibility condition. (In English)

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Let P be a set of prime numbers. For any subset A of P let ΠA denote the product of all primes in A. The set P is said to satisfy condition (*) if $\gcd(\Pi A - \Pi B, \Pi P) = 1$ for all disjoint, non- empty subsets A, B of P. The authors have previously proved [J. Graph Theory 13, No. 5, 593-595 (1989; Zbl 691.05053)] that for all k there exists a set P of k primes satisfying (*). Now let n_k be the smallest ΠP, where P is a set of k primes satisfying (*).

Theorem 1: If P is a set of k primes, $k \geq 2$, satisfying (*), and p is the smallest prime in P, then

$$k \leq \log_2(p-1) + 2.$$

Further, if P cannot be extended to a set of $k+1$ primes satisfying (*) then

$$k \geq \text{Min}(r : 3^{r-1} - 2^{r-1} \geq p-1) = \text{one of } \lceil \log_3(p-1) \rceil + 1 \quad \text{or} \quad \lceil \log_2(p-1) \rceil + 2.$$

Theorem 2: (a) For $k \geq 2$,

$$(\log_2 n_k)/k^2 > 1 - 2/k.$$

(b) For $\varepsilon > 0$,

$$(\log_2 n_k)/k^2 < \log_2(3 + \varepsilon)$$

for all k sufficiently large.

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