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SOME RESULTS ON σ -DERIVATIONS

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ABSTRACT. Let \mathcal{A} and \mathcal{B} be two Banach algebras and let \mathcal{M} be a Banach \mathcal{B} -bimodule. Suppose that $\sigma : \mathcal{A} \rightarrow \mathcal{B}$ is a linear mapping and $d : \mathcal{A} \rightarrow \mathcal{M}$ is a σ -derivation. We prove several results about automatic continuity of σ -derivations on Banach algebras. In addition, we define a notion for m -weakly continuous linear mapping and show that, under certain conditions, d and σ are m -weakly continuous. Moreover, we prove that if \mathcal{A} is commutative and $\sigma : \mathcal{A} \rightarrow \mathcal{A}$ is a continuous homomorphism such that $\sigma^2 = \sigma$ then $\sigma d\sigma(\mathcal{A}) \subseteq \sigma(Q(\mathcal{A})) \subseteq \text{rad}(\mathcal{A})$.

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