Annals of Functional Analysis

Ann. Funct. Anal. 3 (2012), no. 2, 32–57
ANNALS OF FUNCTIONAL ANALYSIS
ISSN: 2008-8752 (electronic)
URL: www.emis.de/journals/AFA/

STABILITY OF A FUNCTIONAL EQUATION OF WHITEHEAD ON SEMIGROUPS

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Communicated by S.-M. Jung

ABSTRACT. Let S be a semigroup and X a Banach space. The functional equation $\varphi(xyz)+\varphi(x)+\varphi(y)+\varphi(z)=\varphi(xy)+\varphi(yz)+\varphi(xz)$ is said to be stable for the pair (X,S) if and only if $f:S\to X$ satisfying $\|f(xyz)+f(x)+f(y)+f(z)-f(xy)-f(yz)-f(xz)\|\leq\delta$ for some positive real number δ and all $x,y,z\in S$, there is a solution $\varphi:S\to X$ such that $f-\varphi$ is bounded. In this paper, among others, we prove the following results: 1) this functional equation, in general, is not stable on an arbitrary semigroup; 2) this equation is stable on periodic semigroups; 3) this equation is stable on abelian semigroups; 4) any semigroup with left (or right) law of reduction can be embedded into a semigroup with left (or right) law of reduction where this equation is stable. The main results of this paper generalize the works of Jung [J. Math. Anal. Appl. 222 (1998), 126–137], Kannappan [Results Math. 27 (1995), 368–372] and Fechner [J. Math. Anal. Appl. 322 (2006), 774–786].

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Date: Received: 9 February 2012; Accepted: March 12 March 2012.
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2010 *Mathematics Subject Classification.* Primary 39B82; Secondary 46L99.
Key words and phrases. Bimorphism, embedding, free groups, periodic semigroup, stability of functional equation.