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## KREIN SPACE NUMERICAL RANGES: COMPRESSIONS AND DILATIONS

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*Dedicated to Professor T. Ando, in recognition of his outstanding mathematical achievements*

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ABSTRACT. A criterion for the numerical range of a linear operator acting in a Krein space to be a two-component hyperbolic disc is given, using the concept of support function. A characterization of the Krein space numerical range as a union of hyperbolic discs is obtained by a reduction to the two-dimensional case. We revisit a famous result of Ando concerning the inclusion relation  $W(A) \subseteq W(B)$  of the numerical ranges of two operators  $A$  and  $B$  acting in (possibly different) Hilbert spaces, and the condition that  $A$  can be dilated to an operator of the form  $B \otimes I$ . The extension of this result to operators acting in Krein spaces is investigated.

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