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(δ, ε) -DOUBLE DERIVATIONS ON BANACH ALGEBRAS

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ABSTRACT. Let \mathcal{A} be an algebra and let $\delta, \varepsilon : \mathcal{A} \rightarrow \mathcal{A}$ be two linear mappings. A (δ, ε) -double derivation is a linear mapping $d : \mathcal{A} \rightarrow \mathcal{A}$ satisfying $d(ab) = d(a)b + ad(b) + \delta(a)\varepsilon(b) + \varepsilon(a)\delta(b)$ ($a, b \in \mathcal{A}$). We study some algebraic properties of these mappings and give a formula for calculating $d^n(ab)$. We show that if \mathcal{A} is a Banach algebra such that either is semi-simple or every derivation from \mathcal{A} into any Banach \mathcal{A} -bimodule is continuous then every (δ, ε) -double derivation on \mathcal{A} is continuous whenever so are δ and ε . We also discuss the continuity of ε when d and δ are assumed to be continuous.

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