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INFINITE-DIMENSIONAL BICOMPLEX HILBERT SPACES

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ABSTRACT. This paper begins the study of infinite-dimensional modules defined on bicomplex numbers. It generalizes a number of results obtained with finite-dimensional bicomplex modules. The central concept introduced is the one of a bicomplex Hilbert space. Properties of such spaces are obtained through properties of several of their subsets which have the structure of genuine Hilbert spaces. In particular, we derive the Riesz representation theorem for bicomplex continuous linear functionals and a general version of the bicomplex Schwarz inequality. Applications to concepts relevant to quantum mechanics, specifically the bicomplex analogue of the quantum harmonic oscillator, are pointed out.

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