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A GENERAL ITERATIVE ALGORITHM FOR NONEXPANSIVE MAPPINGS IN BANACH SPACES

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ABSTRACT. Let E be a real q-uniformly smooth Banach space whose duality map is weakly sequentially continuous. Let $T : E \to E$ be a nonexpansive mapping with $F(T) \neq \emptyset$. Let $A : E \to E$ be an η -strongly accretive map which is also κ -Lipschitzian. Let $f : E \to E$ be a contraction map with coefficient $0 < \alpha < 1$. Let a sequence $\{y_n\}$ be defined iteratively by $y_0 \in$ $E, y_{n+1} = \alpha_n \gamma f(y_n) + (I - \alpha_n \mu A)Ty_n, n \ge 0$, where $\{\alpha_n\}, \gamma$ and μ satisfy some appropriate conditions. Then, we prove that $\{y_n\}$ converges strongly to the unique solution $x^* \in F(T)$ of the variational inequality $\langle (\gamma f - \mu A)x^*, j(y - x^*) \rangle \le 0, \forall y \in F(T)$. Convergence of the correspondent implicit scheme is also proved without the assumption that E has weakly sequentially continuous duality map. Our results are applicable in l_p spaces, 1 .

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