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Classification of span-symmetric generalized quadrangles of order *s*

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Abstract. A line L of a finite generalized quadrangle $\mathcal{S}$ of order (s,t), $s,t > 1$, is an axis of symmetry if there is a group of full size s of collineations of $\mathcal{S}$ fixing any line which meets L. If $\mathcal{S}$ has two non-concurrent axes of symmetry, then $\mathcal{S}$ is called a span-symmetric generalized quadrangle. We prove the twenty-year-old conjecture that every span-symmetric generalized quadrangle of order (s,s) is classical, i.e. isomorphic to the generalized quadrangle $\mathcal{Q}(4,s)$ which arises from a nonsingular parabolic quadric in $\mathrm{PG}(4,s)$.

1 Statement of the main result

In this paper, we prove the following main result.

Theorem 1.1. *Let $\mathcal{S}$ be a span-symmetric generalized quadrangle of order s, where $s \neq 1$. Then $\mathcal{S}$ is classical, i.e. isomorphic to $\mathcal{Q}(4,s)$.*

This has the following corollary for groups with a 4-gonal basis (as defined in Section 3).

Theorem 1.2. *A finite group is isomorphic to $\mathrm{SL}_2(s)$ for some s if and only if it has a 4-gonal basis.*

2 Notation

A (finite) *generalized quadrangle (GQ) of order* (s,t) is an incidence structure $\mathcal{S} = (P,B,I)$, with point set P, line set B and symmetric incidence relation I, where each point is incident with $t+1$ lines ($t \geqslant 1$), each line is incident with $s+1$ points ($s \geqslant 1$), and if a point p is not incident with a line L, then there is a unique point-line pair (q,M) such that $pIMIqIL$. If $s=t$ we say that $\mathcal{S}$ has *order s*. As a general reference we mention the book by S. E. Payne and J. A. Thas [8], see also [10] and [12] for more recent developments, and [11] and [15] for surveys on generalized polygons.

Points p and q of $\mathcal{S} = (P,B,I)$ are *collinear*, if they are incident with a common line. For $p \in P$, put $p^{\perp} = \{q \in P \mid p,q \text{ are collinear}\}$ (note that $p \in p^{\perp}$). More generally, if $A \subseteq P$, we define $A^{\perp} = \bigcap\{p^{\perp} \mid p \in A\}$. Often we use the dual notion $L^{\perp} = \{M \in B \mid L, M$ are confluent$\}$ for lines L, and $X^{\perp} = \bigcap\{L^{\perp} \mid L \in X\}$ for $X \subseteq B$. If Y is a subset of P or of B, then $Y^{\perp\perp}$ denotes $(Y^{\perp})^{\perp}$.

The *classical GQ* $\mathcal{Q}(d, q)$, $d \in \{3, 4, 5\}$, is the GQ which arises by taking the points and lines of a nonsingular quadric with Witt index 2 (that is, with projective index 1) in the d-dimensional projective space $\mathrm{PG}(d, q)$ over the Galois field $\mathrm{GF}(q)$. Respectively, the orders are $(q, 1)$, (q, q) and (q, q^2).

3 Span-symmetric generalized quadrangles

Suppose L is a line of a GQ $\mathcal{S}$ of order (s, t), $s, t \neq 1$. A *symmetry about* L is an automorphism of the GQ which fixes every line of $L^{\perp}$. The line L is called an *axis of symmetry* if there is a group H of symmetries of size s about L. In such a case, if $M \in L^{\perp} \setminus \{L\}$, then H acts regularly on the points of M not incident with L. We remark that every line of the classical example $\mathcal{Q}(4, s)$ is an axis of symmetry (see 8.7.3 of [8]). If L and M are distinct non-concurrent axes of symmetry, then it is easy to see, by transitivity, that every line of $\{L, M\}^{\perp\perp}$ is an axis of symmetry, and $\mathcal{S}$ is called a *span-symmetric generalized quadrangle (SPGQ)* with *base-span* $\{L, M\}^{\perp\perp}$. In this situation, we will use the following *notation* throughout this paper: the base-span will always be denoted by $\mathcal{L}$. The group which is generated by all the symmetries about the lines of $\mathcal{L}$ is G, and we call this group the *base-group*. This group clearly acts 2-transitively on the lines of $\mathcal{L}$, and fixes every line of $\mathcal{L}^{\perp}$ (see for instance 10.7 of [8]).

Theorem 3.1 (S. E. Payne [7]; see also 10.7.2 of [8]). *If $\mathcal{S}$ is an SPGQ of order s, $s \neq 1$, with base-group G, then G acts regularly on the set of $(s+1)s(s-1)$ points of $\mathcal{S}$ which are not on any line of $\mathcal{L}$.*

Note. There is an analogue of Theorem 3.1 for SPGQ's of order (s, s^2), $s > 1$, see K. Thas [13] and [14].

Let $\mathcal{S}$ be an SPGQ of order $s \neq 1$ with base-span $\mathcal{L}$, and put $\mathcal{L} = \{U_0, \ldots, U_s\}$. The group of symmetries about U_i is denoted by G_i, $i = 0, 1, \ldots, s$, throughout this paper. Then one notes the following properties (see [7] and 10.7.3 of [8]):

1. the groups $G_0, \ldots, G_s$ form a complete conjugacy class in G, and are all of order s, $s \geqslant 2$;
2. $G_i \cap N_G(G_j) = \{\mathbf{1}\}$ for $i \neq j$;
3. $G_i G_j \cap G_k = \{\mathbf{1}\}$ for i, j, k distinct, and
4. $|G| = s^3 - s$.

We say that G is a group with a 4-*gonal basis* $\mathcal{T} = \{G_0, \ldots, G_s\}$ if these four conditions are satisfied.

It is possible to recover the GQ $\mathcal{S}$ of order s from the base-group G starting from 4-gonal bases, see [7] and 10.7.8 of [8], hence

Theorem 3.2 (S. E. Payne [7]; see also 10.7.8 of [8]). *A span-symmetric GQ of order $s \neq 1$ with given base-span $\mathscr{L}$ is canonically equivalent to a group G of order $s^3 - s$ with a 4-gonal basis $\mathscr{T}$.*

Now suppose G is a group of order $s^3 - s$, where s is a power of a prime p, and suppose G has a 4-gonal basis $\mathscr{T} = \{G_0, \dots, G_s\}$. Since the groups G_i all have order s, all these groups are Sylow p-subgroups in G. Since $\mathscr{T}$ is a complete conjugacy class, this means that every Sylow p-subgroup of G is contained in $\mathscr{T}$, and hence G has exactly $s+1$ Sylow p-subgroups. Hence we have proved the following easy but important theorem.

Theorem 3.3. *Suppose G is a group of order $s^3 - s$ with s a power of a prime. Then G can have at most one 4-gonal basis. In particular, if G has a 4-gonal basis, then it is unique.*

As a corollary we obtain

Theorem 3.4. *Suppose $\mathscr{S}$ is a span-symmetric GQ of order s, $s \neq 1$. Then $\mathscr{S}$ is isomorphic to the classical GQ $\mathscr{Q}(4,s)$ if and only if the base-group is isomorphic to* $\mathrm{SL}_2(s)$.

Proof. Suppose that the base-group G is isomorphic to $\mathrm{SL}_2(s)$; then s is a power of a prime and hence by Theorem 3.3, $\mathrm{SL}_2(s)$ has at most one 4-gonal basis. Now consider a $\mathscr{Q}(4,s)$ and suppose L and M are non-concurrent lines of $\mathscr{Q}(4,s)$. Then L and M are axes of symmetry, and hence $\mathscr{Q}(4,s)$ is span-symmetric for the base-span $\{L, M\}^{\perp\perp}$. In this case, the base-group is isomorphic to $\mathrm{SL}_2(s)$ (see e.g. [7]), which proves that $\mathrm{SL}_2(s)$ has a 4-gonal basis, necessarily unique by Theorem 3.3. Hence, by Theorem 3.2, there is only one GQ which can arise from $\mathrm{SL}_2(s)$ using 4-gonal bases and this is $\mathscr{Q}(4,s)$, hence $\mathscr{S} \cong \mathscr{Q}(4,s)$. □

It was conjectured in 1980 by S. E. Payne that a span-symmetric generalized quadrangle of order $s > 1$ is always classical, i.e. isomorphic to the GQ $\mathscr{Q}(4,s)$ arising from a quadric. There was a "proof" of this theorem as early as in 1981 by Payne in [7], but later on, it was noticed by the author himself that there was a mistake in the proof. The paper was very valuable however, since the author introduced there the 4-gonal bases and proved for instance Theorem 3.2 and Theorem 5.1 (see below).

4 The base-group G

From now on, we denote by N the kernel of the action of G on the lines of $\mathscr{L}$. The notation of Section 3 will be used freely. The following result is crucial:

Theorem 4.1. *Suppose $\mathscr{S}$ is a span-symmetric generalized quadrangle of order (s,t), $s, t \neq 1$, with base-span $\mathscr{L}$ and base-group G. Then G/N acts as a sharply 2-transitive group on $\mathscr{L}$, or is isomorphic, as a permutation group, to one of the following:*

(a) $\mathrm{PSL}_2(s)$, (b) *the Ree group* $\mathrm{R}(\sqrt[3]{s})$, (c) *the Suzuki group* $\mathrm{Sz}(\sqrt{s})$, (d) *the unitary group* $\mathrm{PSU}_3(\sqrt[3]{s^2})$, *each with its natural action of degree* $s+1$.

Proof. The group G (and hence also G/N) is doubly transitive on $\mathscr{L}$, and for every $L \in \mathscr{L}$ the full group of symmetries about L, which acts regularly on $\mathscr{L}\setminus\{L\}$, is a normal subgroup of the stabilizer of L in G. This means that $(\mathscr{L}, G/N)$ is a split BN-pair of rank 1. All finite groups with a split BN-pair of rank 1 have been classified by Shult [9] and Hering, Kantor and Seitz [3], without using the classification of the finite simple groups. Their results give the above list of possibilities for G/N, noting that G/N is generated by the normal subgroups mentioned above. □

Lemma 4.2. *G is a perfect group if G/N does not act sharply 2-transitively on $\mathscr{L}$.*

Proof. Suppose G/N does not act sharply 2-transitively on $\mathscr{L}$. By Theorem 4.1, G/N is isomorphic to one of the following: (a) $\mathrm{PSL}_2(s)$; (b) $\mathrm{R}(\sqrt[3]{s})$; (c) $\mathrm{Sz}(\sqrt{s})$; (d) $\mathrm{PSU}_3(\sqrt[3]{s^2})$. All these groups are perfect groups.* Assume that G is distinct from its derived group G'. Then since G/N is a perfect group, we have that $(G/N)' = G'N/N = G/N$, and hence $G'N = G$. First suppose we are in Case (a). If s is even, then $|G| = |\mathrm{PSL}_2(s)|$, and thus $|N| = \{\mathbf{1}\}$. So in that case $G = G'$, a contradiction. If s is odd, then G' is a subgroup of G of index 2. It follows that G and G' have exactly the same Sylow p-subgroups, with s a power of the odd prime p. Since here G is generated by its Sylow p-subgroups (by the definition of the base-group G), we infer that $G = G'$, a contradiction. Hence G is perfect.

Now suppose we are in Case (b) or (c). Then $|N| = \frac{s-1}{s^n-1}$ with $n \in \{1/2, 1/3\}$, and hence $|N|$ and s are mutually coprime since $s-1$ and s are mutually coprime. Hence s is a divisor of $|G'|$, since $|G| = \frac{|G'|\times|N|}{|G'\cap N|}$. Thus G and G' have precisely the same Sylow p-subgroups, with s a power of the prime p. Since here G is generated by its Sylow p-subgroups, we conclude that $G = G'$, a contradiction. Finally, assume that we are in the last case. Then $|N| = \frac{(3,\sqrt[3]{s}+1)(s-1)}{\sqrt[3]{s^2}-1}$, and thus it is clear that $|N|$ and s are mutually coprime. The same argument as before yields that $|G'| \equiv 0 \bmod s$, and hence that $G = G'$, a contradiction. Consequently G is perfect. □

Remark 4.3. For $s = 2$ the GQ is isomorphic to $\mathscr{Q}(4,2)$ (6.1 of [8]). In this case $G = G/N \cong S_3$ acts sharply 2-transitively on $\mathscr{L}$.

Lemma 4.4. *N is in the center of G.*

Proof. Clearly N is a normal subgroup of G. Let H be the full group of symmetries about an arbitrary line of $\mathscr{L}$. Then N and H normalize each other, and hence they commute. □

* With the exception of $\mathrm{R}(3)$; for this case see the following paper by W. M. Kantor (Editor's note).

Lemma 4.5. *If $\mathcal{S}$ is an SPGQ of order $s \neq 1$ with base-group G and base-span $\mathcal{L}$, then G/N acts either as* $\mathrm{PSL}_2(s)$ *or as a sharply* 2*-transitive group on the lines of $\mathcal{L}$.*

Proof. Assume by way of contradiction that G/N does not act as $\mathrm{PSL}_2(s)$ or a sharply 2-transitive group on the lines of $\mathcal{L}$. First of all, G is a perfect group, and since N is in the center of G, the group G is a perfect central extension of the group G/N which acts on $\mathcal{L}$. The perfect group G/N has a universal central extension $\overline{G/N}$, and $\overline{G/N}$ contains a central subgroup F such that $\overline{G/N}/F \cong G$, see e.g. [6]. We now look at the possible cases.

If $G/N \cong \mathrm{Sz}(\sqrt{s})$, and if $s > 8^2$, then N must be trivial since in that case the Suzuki group has a trivial universal central extension (i.e. $\overline{G/N} \cong G/N$) by [2] p. 302, an impossibility since the orders of G and $\mathrm{Sz}(\sqrt{s})$ are not the same if $s > 8^2$. Suppose that $s = 8^2$. Then by [2] p. 302 any perfect central extension H of $\mathrm{Sz}(8)$ satisfies $|H| = 2^k|\mathrm{Sz}(8)|$ for some $k \in \{0, 1, 2\}$. None of these cases occurs since $|G| = (64)^3 - 64 = 262080$ and since $|\mathrm{Sz}(8)| = 29120$.

If $G/N \cong \mathrm{R}(\sqrt[3]{s})$, then we have exactly the same situation as in the preceding case, compare [2] p. 302, hence this case is excluded as well.

Finally, assume that $G/N \cong \mathrm{PSU}_3(\sqrt[3]{s^2})$. The universal central extension of $\mathrm{PSU}_3(\sqrt[3]{s^2})$ is known to be $\mathrm{SU}_3(\sqrt[3]{s^2})$, see [2] p. 302, and also, we know that $|\mathrm{SU}_3(\sqrt[3]{s})| = (3, \sqrt[3]{s} + 1)|\mathrm{PSU}_3(\sqrt[3]{s})| = (s + 1)s(\sqrt[3]{s^2} - 1)$ ([4], pages 420 and 421). This provides us with a contradiction since $s > 1$, hence $s - 1 > \sqrt[3]{s^2} - 1$. □

Lemma 4.6. *If G/N acts as* $\mathrm{PSL}_2(s)$*, then $G \cong$* $\mathrm{SL}_2(s)$ *and $\mathcal{S}$ is classical.*

Proof. The universal central extension of $\mathrm{PSL}_2(s)$ is $\mathrm{SL}_2(s)$, except in the cases $s = 4$ and $s = 9$, compare [2] p. 302, and in general $|\mathrm{SL}_2(s)| = (2, s - 1)|\mathrm{PSL}_2(s)| = |G|$, see pages 420 and 421 of [4]. Hence if $s \neq 4, 9$, then G is isomorphic to $\mathrm{SL}_2(s)$, and by Theorem 3.4 $\mathcal{S}$ is classical.

There is a unique GQ of order 4, namely $\mathcal{Q}(4, 4)$, see e.g. 6.3.1 of [8], so $s = 4$ gives no problem; in this case, G is isomorphic to $\mathrm{SL}_2(4)$. Finally, suppose that $s = 9$. Then there is only one possible perfect central extension of $G/N \cong \mathrm{PSL}_2(9)$ with size $9^3 - 9 = 234$, namely $\mathrm{SL}_2(9)$, see [2] p. 302. Hence $G \cong \mathrm{SL}_2(9)$, and by Theorem 3.4 $\mathcal{S}$ is classical and isomorphic to $\mathcal{Q}(4, 9)$. □

5 The sharply 2-transitive case

We recall the following.

Theorem 5.1 (S. E. Payne [7]; see also 10.7.9 of [8]). *Let $\mathcal{S}$ be an SPGQ of order $s \neq 1$, with base-span $\mathcal{L}$. Then every line of $\mathcal{L}^{\perp}$ is an axis of symmetry.*

This theorem thus yields the fact that for any two distinct lines U and V of $\mathcal{L}^{\perp}$, the GQ is also an SPGQ with base-span $\{U, V\}^{\perp\perp}$. The corresponding base-group will be denoted by $G^{\perp}$. It should be emphasized that this property only holds for SPGQ's of order s (see [13]).

Suppose that G/N acts as a sharply 2-transitive group on the lines of $\mathscr{L}$ in the SPGQ $\mathscr{S}$ of order $s > 1$. Since the lines of $\mathscr{L}^{\perp}$ are also axes of symmetry, we can assume that the base-group $G^{\perp}$ corresponding to these lines also acts as a sharply 2-transitive group on $\mathscr{L}^{\perp}$, because otherwise $G^{\perp}$ is isomorphic to $\mathrm{SL}_2(s)$, and then $\mathscr{S}$ is classical by Theorem 3.4. Hence G and $G^{\perp}$ contain normal central subgroups N and $N^{\perp}$, both of order $s-1$, which act trivially on the points of Ω, where Ω is the set of points on the lines of the base-span. Note that G and $G^{\perp}$ act regularly on the points of $\mathscr{S}$ not in Ω by Theorem 3.1.

Let p be a point and L a line of a projective plane Π. Then Π is said to be (p, L)-*transitive* if the group of all collineations of Π with center p and axis L acts transitively on the points, distinct from p and not on L, of any line through p. The following theorem is a step in the Lenz–Barlotti classification of finite projective planes, see e.g. [1] or [16]; it states that the Lenz–Barlotti class III.2 is empty.

Theorem 5.2 (J. C. D. S. Yaqub [17]). *Let Π be a finite projective plane, containing a non-incident point-line pair (x, L) for which Π is (x, L)-transitive, and assume that Π is (y, xy)-transitive for every point y on L. Then Π is Desarguesian.*

Note that every axis of symmetry L is *regular* in the sense of S. E. Payne and J. A. Thas [8, 1.3]; hence there is a projective plane Π_L canonically associated with L as in 1.3.1 of [8].

Theorem 5.3. *Suppose that $\mathscr{S}$ is an SPGQ of order s, where $s \neq 1$, with base-group G and base-span $\mathscr{L}$. Also, let N be the kernel of the action of G on the lines of $\mathscr{L}$, and suppose that G/N acts as a sharply 2-transitive group on the lines of $\mathscr{L}$. Then $\mathscr{S}$ is isomorphic to $\mathscr{Q}(4,2)$ or $\mathscr{Q}(4,3)$.*

Proof. Fix a line L of $\mathscr{L}$, and consider the projective plane Π_L^* of order s, which is the dual of Π_L. Then $\mathscr{L}^{\perp}$ is a point of Π_L^* which is not incident with L as a line of the plane. For convenience, denote this point by p. Now consider the action of N as a collineation group on Π_L^*. Clearly, this action is faithful (recall that N fixes Ω pointwise). Then, as $|N| = s-1$ and as N fixes L pointwise and p linewise, the plane Π_L^* is (p, L)-transitive.

Now fix an arbitrary line U through p in Π_L^*; then U is a line of $\mathscr{L}^{\perp}$. If we interpret the group $G_U^{\perp}$ of all symmetries about U as a collineation group of Π_L^* (this is possible since $G_U^{\perp}$ fixes L), then $G_U^{\perp}$ fixes every line through the point $L \cap U$ of Π_L^*. Suppose r is an arbitrary point of Π_L^* on U and different from $L \cap U$. Then, in the GQ, r is of the form $\{U, U'\}^{\perp\perp}$, with U' some line of $L^{\perp}$ which does not meet U. It is clear that for any symmetry θ about U we have $(\{U, U'\}^{\perp\perp})^{\theta} = \{U, U'\}^{\perp\perp}$, and thus any element of $G_U^{\perp}$ as a collineation of Π_L^* fixes every point on the line U. From the fact that $|G_U^{\perp}| = s$, and that distinct elements of $G_U^{\perp}$ induce distinct collineations of Π_L^*, it follows that Π_L^* is $(U \cap L, U)$-transitive. Hence by Theorem 5.2 the plane Π_L^* is Desarguesian.

Now consider the action of the groups $G_V^{\perp}$ on Π_L^*, with $V \in \mathscr{L}^{\perp}$. Then $G_V^{\perp}$ fixes the line L and the point $V \cap L$ and acts regularly on the other points of L. The group

$G^{\perp} = \langle G_V^{\perp} \mid V \in \mathscr{L}^{\perp} \rangle$, as a collineation group of the plane, induces a sharply 2-transitive permutation group on the points of L by our hypothesis. But since the plane Π_L^* is Desarguesian, we also know that the groups $G_V^{\perp}$, as collineation groups of the plane, generate a $\mathrm{PSL}_2(s)$ on L, and so, as $|\mathrm{PSL}_2(s)| = |G^{\perp}| = s^3 - s$ ($G^{\perp}$ acts faithfully on Π_L^*), we have that $s \in \{2, 3\}$.

Now suppose that $s = 2$. Then $\mathscr{S}$ is isomorphic to the unique GQ of order 2, namely the classical $\mathscr{Q}(4,2)$ (see 6.1 of [8]). Finally, suppose that $s = 3$. Then $\mathscr{S} \cong \mathscr{Q}(4,3)$ (see 3.3.1 and 6.2 of [8], and recall that $\mathscr{S}$ has regular lines). □

Note. There is also an elementary group-theoretical proof of the last theorem, as was pointed out to us by W. M. Kantor [5].

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