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WEAK PSEUDO-COMPLEMENTATIONS ON ADL'S

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Abstract. The notion of an Almost Distributive Lattice (abbreviated as ADL) was introduced by U. M. Swamy and G. C. Rao [6] as a common abstraction of several lattice theoretic and ring theoretic generalization of Boolean algebras and Boolean rings. In this paper, we introduce the concept of weak pseudo-complementation on ADL's and discuss several properties of this.

1. Introduction

O. Frink [2] has proved that, in any pseudo-complemented semi lattice S, the set $S^* = \{a^* \mid a \in S\}$ becomes a Boolean algebra which is a sub semi lattice of S. K. B. Lee [3] has proved that the class of distributive pseudo-complemented lattice is equationally definable and hence a variety (a class which is closed under the formation of subalgebras, homomorphic images and products). Further, U. M. Swamy, G. C. Rao and G. N. Rao [7] have introduced the notion of pseudo-complementation on an Almost Distributive Lattice (ADL) and proved that the class of pseudo-complemented ADL's is also equationally definable. Here, we introduce the concept of weak pseudo-complementation on an ADL and discuss several properties of ADL's with weak pseudo-complementation. In particular, we prove that an ADL is pseudo-complemented if and only if it is weakly pseudo-complemented, even though a weak pseudo-complementation need not be a pseudo-complementation in general.

2. Preliminaries

We first recall certain elementary definitions and results concerning Almost Distributive Lattices. These are collected from [6] and [7].

Definition 2.1. An algebra $A = (A, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is called an Almost Distributive Lattice (abbreviated as ADL) if it satisfies the following identities

(1) $0 \wedge a \approx 0$;
(2) $a \vee 0 \approx a$;
(3) $a \wedge (b \vee c) \approx (a \wedge b) \vee (a \wedge c)$;

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(4) $(a \vee b) \wedge c \approx (a \wedge c) \vee (b \wedge c)$;
(5) $a \vee (b \wedge c) \approx (a \vee b) \wedge (a \vee c)$;
(6) $(a \vee b) \wedge b \approx b$.

Any distributive lattice bounded below is an ADL, where 0 is the smallest element. Also, a commutative regular ring $(R, +, \cdot, 0, 1)$ with unity can be made into an ADL by defining the operations $\wedge$ and $\vee$ on R by

$$a \wedge b = a_0 b \quad \text{and} \quad a \vee b = a + b - a_0 b\,,$$

where, for any $a \in R$, a_0 is the unique idempotent in R such that $aR = a_0 R$ and 0 is the additive identity in R. Further any non empty set X can be made into an ADL by fixing an arbitrarily choosen element 0 in X and by defining the operations $\wedge$ and $\vee$ on by X by

$$a \wedge b = \begin{cases} 0, & \text{if } a = 0 \\ b, & \text{if } a \neq 0 \end{cases} \quad \text{and} \quad a \vee b = \begin{cases} b, & \text{if } a = 0 \\ a, & \text{if } a \neq 0. \end{cases}$$

This ADL $(X, \wedge, \vee, 0)$ is called a discrete ADL. An ADL A is said to be associate ADL if the operation $\vee$ on A is associate. Through out this paper, by an ADL we mean an associate ADL only.

Definition 2.2. Let $A = (A, \wedge, \vee, 0)$ be an ADL. For any a and $b \in A$, define

$$a \leq b \quad \text{if and only if} \quad a = a \wedge b\,, \quad \text{this is equivalent to} \quad a \vee b = b\,.$$

Then $\leq$ is a partial order on A.

Theorem 2.3. *The following hold for any a, b and c in an ADL $A = (A, \wedge, \vee, 0)$.*
(1) $a \wedge 0 = 0 = 0 \wedge a$ *and* $a \vee 0 = a = 0 \vee a$;
(2) $a \wedge a = a = a \vee a$;
(3) $a \wedge b \leq b \leq b \vee a$;
(4) $a \wedge b = a \Leftrightarrow a \vee b = b$;
(5) $a \vee b = a \Leftrightarrow a \wedge b = b$;
(6) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$;
(7) $a \vee (b \vee a) = a \vee b$;
(8) $a \leq b \Rightarrow a \wedge b = a = b \wedge a \ \Leftrightarrow a \vee b = b = b \vee a$;
(9) $(a \wedge b) \wedge c = (b \wedge a) \wedge c$;
(10) $(a \vee b) \wedge c = (b \vee a) \wedge c$;
(11) $a \wedge b = b \wedge a \Leftrightarrow a \vee b = b \vee a$;
(12) $a \wedge b = \inf\{a, b\} \Leftrightarrow a \wedge b = b \wedge a \Leftrightarrow a \vee b = \sup\{a, b\}$.

Definition 2.4. A non empty subset I of an ADL A is said to be an ideal of A if $a \vee b \in I$ for all $a \in I$ and $b \in I$ and $x \wedge a \in I$ for all $x \in I$ and $a \in A$.

It follows as a consequence that $a \wedge x \in I$ for all $x \in I$ and $a \in A$. For any $X \subseteq A$, the smallest ideal of A containing X is called the ideal generated by X and is denoted by $\langle X]$. If $X = \{x\}$, we simply write $\langle x]$ for $\langle \{x\}]$. We have the following for any $X \subseteq A$ and $x \in A$.

$$\langle X] = \Big\{ \big(\bigvee_{i=1}^{n} x_i \big) \wedge a \mid n \geq 0,\ x_i \in X \text{ and } a \in A \Big\}$$

$$\text{and} \qquad \langle\, x] = \langle\, \{x\}] = \{x \wedge a \mid a \in A\} = \{y \in A \mid x \wedge y = y\}\,,$$

$\langle\, x]$ is called the principal ideal generated by x.

3. Weak pseudo-complementations on ADL's

The concept of pseudo-complementation on an ADL was first introduced by U. M. Swamy, G. C. Rao and G. N. Rao [7] and they have proved that the class of pseudo-complemented ADL's is an equationally definable class. Also, for any ADL A in this class, they have exhibited a one-to-one correspondence between maximal elements in A and pseudo-complementations on A. We prove certain important properties of pseudo-complemented ADL's by making a slight modifications of the definition of pseudo-complementations given in [7].

First, let us recall that, for any elements a and b in an ADL A, $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$ (since $a \wedge b \wedge a = b \wedge a$). For any subset S of A, let

$$S^* = \{a \in A \mid a \wedge s = 0 \quad \text{for all} \quad s \in S\}\,.$$

Then S^* is always an ideal of A for all $S \subseteq A$. It can be easily proved that $S^* = \langle\, S]^*$. For any $a \in A$, we have

$$\langle\, a]^* = \{a\}^* = \{x \in A \mid a \wedge x = 0\} = \{x \in A \mid x \wedge a = 0\}\,.$$

Definition 3.1. Let $A = (A, \wedge, \vee, 0)$ be an ADL. A mapping $a \mapsto a^*$ of A into itself is called a weak pseudo-complementation on A if

$$a \wedge b = 0 \Leftrightarrow a^* \wedge b = b$$

for any a and $b \in A$.

The following is a straight forward verification.

Theorem 3.2. *The following are equivalent to each other for any mapping $a \mapsto a^*$ of an ADL A into itself.*

(1) $a \mapsto a^*$ *is a weak pseudo-complementation on A;*

(2) $\{a\}^* = \langle\, a^*]$ *for any $a \in A$;*

(3) *For any $a \in A$, $a \wedge a^* = 0$; and $a \wedge b = 0 \Rightarrow a^* \wedge b = b$ for any $b \in A$.*

Definition 3.3. An ADL A is said to be weakly pseudo-complemented if there is a weak pseudo-complementation $a \mapsto a^*$ on A.

The following is an immediate consequence of Theorem 3.2 and the axiom of choice.

Corollary 3.4. *An ADL A is weakly pseudo-complemented if and only if $\{a\}^*$ is a principal ideal for any $a \in A$.*

Note that a principal ideal in an ADL may have more than one generators, unlike the case of a lattice in which any principal ideal has a unique generator. However, for any a and b in an ADL, we have

$$\begin{aligned}\langle\, a] = \langle\, b] &\Leftrightarrow a \wedge b = b \quad \text{and} \quad b \wedge a = a \\ &\Leftrightarrow a \vee b = a \quad \text{and} \quad b \vee a = b\end{aligned}$$

and we denote this situation by writing $a \sim b$ and calling a and b as associates to each other. In this context, we have the following.

Theorem 3.5. *Let $a \mapsto a^*$ and $a \mapsto a^+$ be two weak pseudo-complementations on an ADL A. Then the following hold for any a and $b \in A$.*

(1) $a^* \sim a^+$;

(2) $a^{*+} \sim a^{++}$;

(3) $a^* \sim b^* \Leftrightarrow a^+ \sim b^+$;

(4) $a^* = 0 \Leftrightarrow a^+ = 0$;

(5) $a^* \wedge 0^+ \sim a^+$;

(6) $a^* \vee a^{**} \sim 0^* \Leftrightarrow a^+ \vee a^{++} \sim 0^+$.

Proof.

(1) We have $\langle\, a^*] = \{a\}^* = \langle\, a^+]$ (by Theorem 3.2) and therefore $a^* \sim a^+$.

(2) We have $\langle\, a^{*+}] = \{a^*\}^* = \langle\, a^*]^* = \langle\, a^+]^* = \{a^+\}^* = \langle\, a^{++}]$ and therefore $a^{*+} \sim a^{++}$.

(3) $a^* \sim b^* \Leftrightarrow \langle\, a^*] = \langle\, b^*]$
$\Leftrightarrow \{a\}^* = \{b\}^* \Leftrightarrow \langle\, a^+] = \langle\, b^+] \Leftrightarrow a^+ \sim b^+$.

(4) $a^* = 0 \Leftrightarrow \langle\, a^*] = \{0\}$
$\Leftrightarrow \{a\}^* = \{0\} \Leftrightarrow \langle\, a^+] = \{0\} \Leftrightarrow a^+ = 0$.

(5) $\langle\, a^+] = \{a\}^* \cap A = \langle\, a^*] \cap \{0\}^* = \langle\, a^*] \cap \langle\, 0^+] = \langle\, a^* \wedge 0^+]$
and therefore $a^* \wedge 0^+ \sim a^+$.

(6) $a^* \vee a^{**} \sim 0^* \Rightarrow a^+ \vee a^{++} \sim a^+ \vee a^{*+} \sim (a^* \wedge 0^+) \vee (a^{**} \wedge 0^+)$
$= (a^* \vee a^{**}) \wedge 0^+ = 0^* \wedge 0^+ \sim 0^+$.

□

Since $a \sim b$ implies $a = b$ for any elements a and b in a lattice, we have the following.

Corollary 3.6. *Any distributive lattice with 0 has at most one weak pseudo-complementation.*

Let us recall that an element m in an ADL A is maximal in $(A, \leq)$ if and only if $m \wedge a = a (\Leftrightarrow m = m \vee a)$ for all $a \in A$, which is equivalent to saying that $\langle\, m] = A$.

Theorem 3.7. *Let $a \mapsto a^*$ be a weak pseudo-complementation on an ADL A. Then the following hold for any $a \in A$ and $b \in A$.*

(1) 0^* *is a maximal element in* A;

(2) m *is maximal in* $A \Rightarrow m^* = 0$;

(3) $0^{**} = 0$;

(4) $a^* \wedge a = 0$;
(5) $a^{**} \wedge a = a$;
(6) $a \wedge b = 0 \Leftrightarrow a^{**} \wedge b = 0 \Leftrightarrow a \wedge b^{**} = 0 \Leftrightarrow a^{**} \wedge b^{**} = 0$;
(7) $a^* \sim a^{***}$;
(8) $a^* = 0 \Leftrightarrow a^{**}$ *is maximal;*
(9) $a = 0 \Leftrightarrow a^{**} = 0$;
(10) $(a \vee b)^* \sim a^* \wedge b^*$.

Proof.

(1) $\langle\, 0^*] = \{0\}^* = A$ and hence 0^* is maximal.

(2)
$$\begin{aligned} m \text{ is a maximal in } A &\Rightarrow \langle\, m] = A \\ &\Rightarrow \langle\, m^*] = \langle\, m]^* = A^* = \{0\} \\ &\Rightarrow m^* = 0\,. \end{aligned}$$

(3) $\langle\, 0^{**}] = \{0^*\}^* = A^* = \{0\}$ and therefore $0^{**} = 0$.

(4) Since $a \wedge a^* = 0$, we have $a^* \wedge a = a^* \wedge a \wedge a = a \wedge a^* \wedge a = 0 \wedge a = 0$.

(5) Since $a^* \wedge a = 0$, we have $a \in \{a^*\}^* = \langle\, a^{**}]$ and hence $a^{**} \wedge a = a$.

(6)
$$\begin{aligned} a \wedge b = 0 &\Rightarrow a^* \wedge b = b \\ &\Rightarrow a^{**} \wedge b = a^{**} \wedge (a^* \wedge b) = 0 \wedge b = 0 \\ &\Rightarrow b \wedge a^{**} = 0 \\ &\Rightarrow b^{**} \wedge a^{**} = 0 \\ &\Rightarrow a^{**} \wedge b^{**} = 0 \\ &\Rightarrow a \wedge b = a^{**} \wedge a \wedge b^{**} \wedge b \\ &\quad = a^{**} \wedge b^{**} \wedge a \wedge b = 0 \wedge a \wedge b = 0\,. \end{aligned}$$

(7) By (6), we have $\{a\}^* = \{a^{**}\}^*$ and therefore $\langle\, a^*] = \langle\, a^{***}]$ which implies that $a^* \sim a^{***}$.

(8) This follows from (1), (2) and (7) (Note that $x \sim 0 \Rightarrow x = 0$).

(9) Follows from (1), (2) and (5).

(10) We have
$$\begin{aligned} \langle\, a^* \wedge b^*] &= \langle\, a^*] \cap \langle\, b^*] \\ &= \{a\}^* \cap \{b\}^* \\ &= \{a \vee b\}^* \quad \text{(by the distributivity of } \wedge \text{ over } \vee) \\ &= \langle\, (a \vee b)^*] \end{aligned}$$
and therefore $(a \vee b)^* \sim a^* \wedge b^*$.

□

Theorem 3.8. *Let A be an ADL and $a \mapsto a^*$ be a weak pseudo-complementation on A. Then the following hold for any a and $b \in A$.*
(1) $a \sim b \Rightarrow a^* \sim b^*$;
(2) $(a \wedge b)^* \sim (b \wedge a)^*$;
(3) $(a \vee b)^* \sim (b \vee a)^*$;
(4) $(a \wedge b)^* \wedge a^* = a^*$;
(5) $(a \wedge b)^* \wedge b^* = b^*$;

(6) $(a \wedge b)^{**} \sim a^{**} \wedge b^{**}$.

Proof. First, let us recall that $S^* = \langle\, S]^*$ for any $S \subseteq A$ and, in particular, $\{a\}^* = \langle\, a]^*$ for any $a \in A$.

(1) $a \sim b \Rightarrow \langle\, a] = \langle\, b] \Rightarrow \langle\, a]^* = \langle\, b]^* \Rightarrow \{a\}^* = \{b\}^*$
$\Rightarrow \langle\, a^*] = \langle\, b^*] \Rightarrow a^* \sim b^*$.

(2) For any $c \in A$, we have $a \wedge b \wedge c = 0 \Leftrightarrow b \wedge a \wedge c = 0$ and therefore $\langle\, a \wedge b]^* = \langle\, b \wedge a]^*$. This implies that $\langle\, (a \wedge b)^*] = \langle\, (b \wedge a)^*]$ and hence $(a \wedge b)^* \sim (b \wedge a)^*$.

(3) This is similar to (2), since $(a \vee b) \wedge c = (b \vee a) \wedge c$.

(4) Since $(a \wedge b) \wedge a^* = b \wedge a \wedge a^* = b \wedge 0 = 0$, we get that $(a \wedge b)^* \wedge a^* = a^*$.

(5) Since $(a \wedge b) \wedge b^* = 0$, we have $(a \wedge b)^* \wedge b^* = b^*$.

(6) We have $a \wedge b \wedge (a \wedge b)^* = 0 = b \wedge a \wedge (a \wedge b)^*$. By repeated use of 3.7(6), we get that $a^{**} \wedge b^{**} \wedge (a \wedge b)^* = 0$.

$$\therefore\ (a \wedge b)^* \wedge a^{**} \wedge b^{**} = 0$$

$$\therefore\ (a \wedge b)^{**} \wedge a^{**} \wedge b^{**} = a^{**} \wedge b^{**}\,. \tag{3.1}$$

On the other hand, we have $(a \wedge b) \wedge b^* = 0$ and hence $\big($again by 3.7(6)$\big)$, $(a \wedge b)^{**} \wedge b^* = 0$.

$$\therefore\ b^* \wedge (a \wedge b)^{**} = 0$$

$$\therefore\ b^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**}$$

Similarly $$a^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**}$$

$$\therefore\ a^{**} \wedge b^{**} \wedge (a \wedge b)^{**} = (a \wedge b)^{**} \tag{3.2}$$

By (3.1) and (3.2), we get that $(a \wedge b)^{**} \sim a^{**} \wedge b^{**}$. □

4. Pseudo-complementations on ADL'S

For any weak pseudo-complementation $*$ on an ADL A, Theorem 3.7(10) gives us that $(a \vee b)^*$ and $a^* \wedge b^*$ are associates to each other, for any a and b in A. In this context, let us recall the following from [7].

Definition 4.1. A weak pseudo-complementation $*$ on an ADL A is called a pseudo-complementation if

$$(a \vee b)^* = a^* \wedge b^* \quad \text{for all} \quad a \quad \text{and} \quad b \in A\,.$$

A is said to be pseudo-complemented if there is a pseudo-complementation on A.

For any elements a and b in a lattice, we have $a \wedge b = b \wedge a$ and hence $a \sim b \Rightarrow a = b$. This together with 3.7(10) implies the following.

Theorem 4.2. *Let $L = (L, \wedge, \vee, 0)$ be a distributive lattice with smallest element 0. Then any weak pseudo-complementation on L is a pseudo-complementation.*

The above theorem is not valid for a general ADL. For, consider the example given in the following.

Example 4.3. Let $A = \{0, 1, 2\}$ be the 3-element discrete ADL with 0 as the zero element and $A^3 = A \times A \times A$ be the product ADL whose operations are defined coordinate-wise. For any $a \in A^3$, let $|a|$ be the number of non zero coordinates of a. If $0 \neq a = (a_1, a_2, a_3) \in A^3$, define $a^* = (a_1^*, a_2^*, a_3^*)$, where

$$a_i^* = \begin{cases} 0\,, & \text{if} \quad a_i \neq 0 \\ 1\,, & \text{if} \quad a_i = 0 \quad \text{and} \quad |a| = 1 \\ 2\,, & \text{if} \quad a_i = 0 \quad \text{and} \quad |a| > 1 \end{cases}$$

and define $0^* = (2, 2, 2)$. For example, $(1, 0, 0)^* = (0, 1, 1)$, $(1, 2, 0)^* = (0, 0, 2)$ and $(2, 0, 1)^* = (0, 2, 0)$. It can be easily checked that $a \mapsto a^*$ is a weak pseudo-complementation on A^3. But this is not a pseudo-complementation; for, let

$$a = (1, 0, 0) \quad \text{and} \quad b = (0, 1, 0)\,.$$
$$\text{Then} \quad a \vee b = (1, 1, 0) \quad \text{and} \quad (a \vee b)^* = (0, 0, 2)\,.$$
$$\text{But} \quad a^* = (0, 1, 1) \quad \text{and} \quad b^* = (1, 0, 1)$$
$$\text{and hence} \quad a^* \wedge b^* = (0, 0, 1) \neq (a \vee b)^*\,.$$

Even though a particular weak pseudo-complementation need not be a pseudo-complementation, it induces one such. This is proved in the following.

Theorem 4.4. *Let $A = (A, \wedge, \vee, 0)$ be an ADL. Then A is weakly pseudo-complemented if and only if it is pseudo-complemented.*

Proof. Suppose that $*$ is a weak pseudo-complementation on A. Choose a maximal element m in A (A has one such; for example, 0^* is maximal). For any $a \in A$, define $a^+ = a^* \wedge m$. Then $a \wedge a^+ = a \wedge a^* \wedge m = 0 \wedge m = 0$ and, for any $b \in A$,

$$a \wedge b = 0 \;\Rightarrow\; a^* \wedge b = b \;\Rightarrow\; a^+ \wedge b = a^* \wedge m \wedge b = a^* \wedge b = b\,.$$

Thus $a \mapsto a^+$ is a weak pseudo-complementation on A. Also, for any a and $b \in A$, we have $(a \vee b)^+ \sim a^+ \wedge b^+$ (by Theorem 3.7(10)). Since $x^+ \leq m$ for all $x \in A$, we have that m is an upper bound of $(a \vee b)^+$ and $a^+ \wedge b^+$. This implies that

$$(a \vee b)^+ = (a^+ \wedge b^+) \wedge (a \vee b)^+ = (a \vee b)^+ \wedge (a^+ \wedge b^+) = a^+ \wedge b^+\,.$$

Thus $a \mapsto a^+$ is a pseudo-complementation on A and hence A is pseudo-complemented. The converse is trivial. □

Definition 4.5. Let A be an ADL and $PC(A)$ and $WPC(A)$ be respectively the sets of pseudo-complementations and weak pseudo-complementations on A. Any $*$ and $+$ in $WPC(A)$ are said to be equivalent (and denote this by $* \approx +$) if $0^* = 0^+$. Then clearly $\approx$ is an equivalence relation on $WPC(A)$.

The proof of Theorem 4.4 suggests the following, whose proof is a straight forward verification.

Theorem 4.6. *Let $*$ be weak pseudo-complementation on an ADL A. For any $a \in A$, define $a^{\overline{*}} = a^* \wedge 0^*$. Then $\overline{*}$ is a pseudo-complementation on A.*

Theorem 4.7. *For any ADL A, the correspondence $* \mapsto \overline{*}$ induces a bijection of $WPC(A)/_{\approx}$ onto $PC(A)$.*

Proof. First we observe that, for any $*$ in $PC(A)$,

$$a^{\overline{*}} = a^* \wedge 0^* = (a \vee 0)^* = a^* \quad \text{for all} \quad a \in A$$

and hence $\overline{*} = *$. This implies that $* \mapsto \overline{*}$ is a surjection correspondence. Also, for any $*$ and $+$ in $PC(A)$,

$$\begin{aligned} * \approx + \Rightarrow 0^* &= 0^+ \\ \Rightarrow a^{\overline{*}} &= a^* \wedge 0^* = 0^+ \wedge a^* \wedge 0^+ \qquad (\text{since } \ 0^+ \ \text{ is maximal}) \\ &= a^* \wedge 0^* = a^* \wedge 0^+ \wedge 0^+ = a^+ \wedge 0^+ \qquad \big(\text{by } 3.5(5)\big) \\ &= a^{\overline{+}} \quad \text{for all} \quad a \in A \\ \Rightarrow \ \overline{*} &= \overline{+}. \end{aligned}$$

Also, $\overline{*} = \overline{+} \ \Rightarrow \ 0^{\overline{*}} = 0^{\overline{+}} \ \Rightarrow \ 0^* \wedge 0^* = 0^+ \wedge 0^+ \ \Rightarrow \ 0^* = 0^+ \ \Rightarrow * \approx +$. Thus $* \mapsto \overline{*}$ induces a bijection of $WPC(A)$ onto $PC(A)$. □

Corollary 4.8. *Let A be a pseudo-complemented ADL. Then $* \mapsto 0^*$ induces a bijection of $WPC(A)/_{\approx}$ onto the set $M(A)$ of all maximal elements of A and therefore $PC(A)$ is bijective with $M(A)$.*

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