ON THE INVARIANCE PROPERTY OF THE FISHER INFORMATION FOR A TRUNCATED LOGNORMAL DISTRIBUTION (I)

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Abstract.The Fisher information is well known in estimation theory. The objective of this paper is to give some definitions and properties for the truncated lognormal distributions. Then, in the next paper ([2]), we shall determine some invariance properties of Fisher's information in the case of these distributions.

Keywords: Fisher information, lognormal distribution, truncated distribution, invariance property

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1.Normal and lognormal distributions

Let X be a normal distribution with density function

$$f(x; m_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left\{-\frac{1}{2}\left(\frac{x - m_x}{\sigma_x}\right)^2\right\}, x \in \mathbf{R}, \tag{1}$$

where the parameters m_x and σ_x^2 have their usual significance, namely: $m_x = E(X), \sigma_x^2 = Var(X), m_x \in \mathbf{R}, \sigma_x > 0$.

Definition 1.1.If X is normally distributed with meanm_x and variance σ_x^2, then the random variable

$$Y = e^X, or X = \ln Y, Y > 0, \tag{2}$$

is said to be lognormally distributed. The lognormal density function is given by

$$g(y; m_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi}\sigma_x}\frac{1}{y}\exp\left\{-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right\}, y > 0, \tag{3}$$

where

$$E(Y) = m_y = \exp\left(m_x + \frac{\sigma_x^2}{2}\right), Var(Y) = \sigma_y^2 = \exp\{2m_x + \sigma_x^2\}\left(e^{\sigma_x^2-1}\right). \quad (4)$$

REMARK 1.1.From the relations (4), we obtain

$$m_x = \ln\left[\frac{m_y^2}{\sqrt{m_y^2 + \sigma_y^2}}\right], \sigma_x^2 = \ln\left[\frac{m_y^2 + \sigma_y^2}{m_y^2}\right]. \quad (5)$$

2.THE TRUNCATED LOGNORMAL DISTRIBUTIONS

DEFINITION 2.1. We say that the random variable Y has a lognormal distribution truncated to the left at $Y = a, a \in \mathbf{R}$, $a > 0$ and to the right at $Y = b, b \in \mathbf{R}$, $a < b$, denoted by $Y_{a\leftrightarrow b}$, if its probability density function , denoted by $g_{a\leftrightarrow b}(y; m_x, \sigma_y^2)$, is of the form

$$g_{a\leftrightarrow b}(y; m_x, \sigma_x^2) = \begin{cases} \frac{k(a,b)}{\sqrt{2\pi}\sigma}\frac{1}{y}\exp\left\{-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right\} & if\ 0 < a \leq y \leq b, \\ 0 & if\ 0 < y < a \\ 0 & if\ y > b, \end{cases} \quad (6)$$

where the constant

$$k(a,b) = \frac{1}{\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)} \quad (7)$$

results from the conditions

$$\begin{aligned} & 1^0\ g_{a\leftrightarrow b}(y; m_x, \sigma_x^2) \geq 0\ if y \in (0, \infty), \\ & 2^0\ \int_0^{+\infty} g_{a\leftrightarrow b}(y; m_x, \sigma_x^2)dx = 1. \end{aligned} \quad (8)$$

REMARK 2.1.The function

$$(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} \exp\left(-\frac{t^2}{2}\right) dt, \quad (9)$$

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for which we have the following relations

$$\Phi(-\infty) = 0, \quad \Phi(0) = \frac{1}{2}, \quad \Phi(+\infty) = 1, \quad \Phi(-z) = 1 - \Phi(z), \qquad (10)$$

is the normal distribution function corresponding to the standard normal random variable

$$Z = \frac{X - m_x}{\sigma_x}, M(Z) = 0, Var(Z) = 1 \qquad (11)$$

which has the probability density function

$$f(z; 0, 1) = f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right), z \in (-\infty, +\infty). \qquad (12)$$

REMARK 2.2. A such truncated probability distribution can be regarded as a conditional probability distribution in the sense that if X has an unrestricted distribution with the probability density function $g(y; m_x, \sigma_x^2)$, then $g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)$, as de ned above, is the probability density function which governs the behavior of subject to the condition that $X_{a \leftrightarrow b}$ is known to lie in $[a, b]$. Also, in such case, we say that the random variable $X_{a \leftrightarrow b}$ has a bilateral truncated lognormal distribution with the limits a and b.

REMARK 2.3. It is easy to see that from the relations (6), (7) and (10) follows

$$\lim_{a \to 0} g_{a \leftrightarrow b}(y; m_x, \sigma_x^2) = \begin{cases} \frac{1}{\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right)} \cdot g(y; m_x, \sigma_x^2) & if \ 0 < y \leq b \\ 0 & if \ y > b, \end{cases} \qquad (13)$$

$$\lim_{b \to +\infty} g_{a \leftrightarrow b}(y; m_x, \sigma_x^2) = = \begin{cases} \frac{1}{1 - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)} \cdot g(y; m_x, \sigma_x^2) & if \ y \geq a \\ 0 & if \ 0 < y \leq a, \end{cases} \qquad (14)$$

and

$$\lim_{a \to -0, b \to +\infty} g_{a \leftrightarrow b}(y; m_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi}\sigma_x} \frac{1}{y} \exp\left[-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right] \ if \ y > 0, \qquad (15)$$

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where $g_{\to b}(y; m_x, \sigma_x^2)$ is the probability density function then when $Y_{\to b}$ has a lognormal distribution truncated to the right at $Y = b$; $g_{a\leftarrow}(y; m_x, \sigma_x^2)$ is the probability density function then when $Y_{a\leftarrow}$ has a lognormal distribution truncated to the left at $Y = a$ and $g(y; m_x, \sigma_x^2)$ is the probability density function then when Y has an ordinary lognormal distribution.

THEOREM 2.1. If $Y_{a\leftrightarrow b}$ is a random variable which follows a bilateral truncated lognormal distribution, then its expected value $E(Y_{a\leftrightarrow b})$ and second moment $E(Y_{a\leftrightarrow b}^2)$ have the following expressions

$$E(Y_{a\leftrightarrow b}) = \exp\left\{m_x + \frac{\sigma_x^2}{2}\right\} \frac{\left[\left(\frac{\ln b - m_x}{\sigma_x} - \sigma_x\right) - \left(\frac{\ln a - m_x}{\sigma_x} - \sigma_x\right)\right]}{\left[\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)\right]}, \qquad (16)$$

$$E(Y_{a\leftrightarrow b}^2) = \exp\left\{2m_x + 2\sigma_x^2\right\} \frac{\left[\left(\frac{\ln b - m_x}{\sigma_x} - 2\sigma_x\right) - \left(\frac{\ln a - m_x}{\sigma_x} - 2\sigma_x\right)\right]}{\left[\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)\right]}. \qquad (17)$$

Proof. Making use of the definition of $E(Y_{a\leftrightarrow b})$, we obtain

$$E(Y_{a\leftrightarrow b}) = \frac{k(a,b)}{\sqrt{2\pi}\sigma_x} \int_a^b \exp\left\{-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right\} dy. \qquad (18)$$

By making the change of variables

$$t = \frac{\ln y - m_x}{\sigma_x}, \qquad (19)$$

we obtain

$$y = \exp\left\{m_x + \sigma_x t\right\}, dy = \sigma_x \exp\left\{m_x + \sigma_x t\right\} dt, \qquad (20)$$

and (18) can be rewritten as follows

$$E(Y_{a\leftrightarrow b}) = \frac{k(a,b)\exp\left\{m_x\right\}}{\sqrt{2\pi}} \int_{\frac{\ln a - m_x}{\sigma_x}}^{\frac{\ln b - m_x}{\sigma_x}} \exp\left\{-\frac{1}{2}(t^2 - 2\sigma_x t)\right\} dt = \qquad (21)$$

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$$= \frac{k(a,b) \exp\left\{m_x + \frac{\sigma_x^2}{2}\right\}}{\sqrt{2\pi}} \int\limits_{\frac{\ln a - m_x}{\sigma_x}}^{\frac{\ln b - m_x}{\sigma_x}} \exp\left\{-\frac{1}{2}(t - \sigma_x)^2\right\} dt = \qquad (22)$$

$$= \frac{k(a,b) \exp\left\{m_x + \frac{\sigma_x^2}{2}\right\}}{\sqrt{2\pi}} \int\limits_{\frac{\ln a - m_x}{\sigma_x} - \sigma_x}^{\frac{\ln b - m_x}{\sigma_x} - \sigma_x} \exp\left\{-\frac{1}{2}w^2\right\} dw, \qquad (23)$$

if we used a new change of variables, namely $w = t - \sigma_x$.

From (23) we obtain just the form (16) for the expected value of $Y_{a \leftrightarrow b}$.

Using an analogous method we can obtain the expression for the second moment $E(Y_{a \leftrightarrow b}^2)$.

Thus, we have

$$E(Y_{a \leftrightarrow b}^2) = \frac{k(a,b)}{\sqrt{2\pi}\sigma_x} \int\limits_a^b y \exp\left\{-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right\} dy =$$

$$= \frac{k(a,b) \exp\left\{2m_x + 2\sigma_x^2\right\}}{\sqrt{2\pi}} \int\limits_{\frac{\ln a - m_x}{\sigma_x}}^{\frac{\ln b - m_x}{\sigma_x}} \exp\left\{-\frac{1}{2}(t - 2\sigma_x)^2\right\} dt =$$

$$= \frac{k(a,b) \exp\left\{2m_x + 2\sigma_x^2\right\}}{\sqrt{2\pi}} \int\limits_{\frac{\ln a - m_x}{\sigma_x} - 2\sigma_x}^{\frac{\ln b - m_x}{\sigma_x} - 2\sigma_x} \exp\left\{-\frac{1}{2}v^2\right\} dv =$$

$$= \exp\left\{2m_x + 2\sigma_x^2\right\} \frac{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x} - 2\sigma_x\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x} - 2\sigma_x\right)\right]}{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]},$$

if we used the change of variables (19), respectively the change of variables $v = t - 2\sigma_x$.

COROLLARY 2.1. If $Y_{a \leftrightarrow b}$ is a random variable with a lognormal distribution truncated to the left at $Y = a$ and to the right at $Y = b$, then

$$\text{Var}(Y_{a \leftrightarrow b}) = E(Y_{a \leftrightarrow b}^2) - [E(Y_{a \leftrightarrow b})]^2 =$$

$$= \exp\left\{2m_x + 2\sigma_x^2\right\} \frac{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x} - 2\sigma_x\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x} - 2\sigma_x\right)\right]}{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]} -$$

$$- \exp\left\{2m_x + \sigma_x^2\right\} \frac{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x} - \sigma_x\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x} - \sigma_x\right)\right]^2}{\left[\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right) - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]^2}.$$

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COROLLARY 2.2.**For the unilateral truncated lognormal random variables** $Y_{a\leftarrow}$ **and** $Y_{\rightarrow b}$, **respectively for the lognormal random variable** Y **we have the following expected values and variances**

$$E(Y_{a\leftarrow}) = \lim_{b\rightarrow+\infty} E(Y_{a\leftrightarrow b}) = \exp\left\{m_x + \frac{\sigma^2}{2}\right\} \frac{\left[1 - \left(\frac{\ln a - m_x}{\sigma_x} - \sigma_x\right)\right]}{\left[1 - \left(\frac{\ln a - m_x}{\sigma_x}\right)\right]}, \quad (24)$$

$$\mathrm{Var}Y_{a\leftarrow} = \lim_{b\rightarrow+\infty} Var(Y_{a\leftrightarrow b}) =$$
$$= \exp\left\{2m_x + 2\sigma_x^2\right\} \frac{\left[1 - \Phi\left(\frac{\ln a - m_x}{\sigma_x} - 2\sigma_x\right)\right]}{\left[1 - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]} - \exp\left\{2m_x + \sigma_x^2\right\} \frac{\left[1 - \Phi\left(\frac{\ln a - m_x}{\sigma_x} - \sigma_x\right)\right]^2}{\left[1 - \Phi\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]^2},$$

$$\mathrm{E}(Y_{\rightarrow b}) = \lim_{a\rightarrow 0} E(Y_{a\leftrightarrow b}) = \exp\left\{m_x + \frac{\sigma^2}{2}\right\} \frac{\Phi\left(\frac{\ln b - m_x}{\sigma_x} - \sigma_x\right)}{\Phi\left(\frac{\ln b - m_x}{\sigma_x}\right)},$$

$$\mathrm{Var}Y_{\rightarrow b} = \lim_{a\rightarrow 0} Var(Y_{a\leftrightarrow b}) =$$
$$= \exp\left\{2m_x + 2\sigma_x^2\right\} \frac{\Phi\left(\frac{\ln b - m_x}{\sigma_x} - 2\sigma_x\right)}{\Phi(\ln b - m_x \sigma_x)} - \exp\left\{2m_x + \sigma_x^2\right\} \frac{\Phi^2\left(\frac{\ln b - m_x}{\sigma_x} - \sigma_x\right)}{\Phi^2\left(\frac{\ln b - m_x}{\sigma_x}\right)},$$

respectively

$$\mathrm{E}(Y) = m_y = \lim_{a\rightarrow-\infty, b\rightarrow+\infty} E(Y_{a\leftrightarrow b}) = \exp\left(m_x + \frac{\sigma_x^2}{2}\right),$$

$$\mathrm{Var}(Y) = \sigma_y^2 = \lim_{a\rightarrow 0, b\rightarrow\infty} Var(Y_{a\leftrightarrow b}) = \exp\{2m_x + \sigma_x^2\}\left(e^{\sigma_x^2} - 1\right).$$

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3. FISHER'S INFORMATION MEASURES FOR A TRUNCATED LOGNORMAL DISTRIBUTION.

Case: m_x-an unknown parameter,σ_x^2-a known parameter

THEOREM 3.1.**If the random variable** $Y_{a\leftrightarrow b}$ **has a bilateral truncated lognormal distribution,that is its probability density function is of the form**

$$g_{a\leftrightarrow b}(y; m_x, \sigma_x^2) = \begin{cases} \frac{k(a,b)}{\sqrt{2\pi}\sigma}\frac{1}{y}\exp\left\{-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2\right\} & if\ 0 < a \leq y \leq b, \\ 0 & if\ 0 < y < a \\ 0 & if\ y > b, \end{cases} \quad (25)$$

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where

$$k(a,b) = \frac{1}{\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)},$$ (26)

then the Fisher's information measure, about the unknown parameter m_x, has the following form

$$I_{Y_{a \leftrightarrow b}}(m_x) = \frac{1}{\sigma_x^2} - \frac{[f(\ln b; m_x, \sigma_x^2) - f(\ln a; m_x, \sigma_x^2)]^2}{\left[\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)\right]^2} -$$

$$-\frac{[(\ln b - m_x)f(\ln b; m_x, \sigma_x^2) - (\ln a - m_x)f(\ln a; m_x, \sigma_x^2)]}{\sigma_x^2\left[\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)\right]},$$ (27)

where

$$f(\ln a; m_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left\{-\frac{1}{2}\left(\frac{\ln a - m_x}{\sigma_x}\right)^2\right\} \in \mathbf{R}^+$$

$$f(\ln b; m_x, \sigma_x^2) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left\{-\frac{1}{2}\left(\frac{\ln b - m_x}{\sigma_x}\right)^2\right\} \in \mathbf{R}^+.$$

Proof. For a such continuous random variable $Y_{a \leftrightarrow b}$ the Fisher information measure, about the unknown parameter m_x, has the form

$$I_{X_{a \leftrightarrow b}}(m_x) = E\left[\left(\frac{\partial \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x}\right)^2\right] =$$

$$= \int_a^b \left(\frac{\partial \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x}\right)^2 g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)dy =$$

$$= -\int_a^b \frac{\partial^2 \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x^2} g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)dy = -E\left[\frac{\partial^2 \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x^2}\right].$$ (28)

Using (25), we obtain

$\ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2) = - \ln(\sqrt{2\pi}\sigma_x) + \ln k(a,b) - \ln y - \frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x}\right)^2,$

respectively

$$\frac{\partial \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x} = \frac{\partial \ln k(a,b)}{\partial m_x} + \frac{\ln y - m_x}{\sigma_x^2} = \frac{1}{k(a,b)}\frac{\partial k(a,b)}{\partial m_x} + \frac{\ln y - m_x}{\sigma_x^2}.$$
(29)

Because

$$\frac{\partial k(a,b)}{\partial m_x} = \frac{\partial}{\partial m_x}\left[\frac{1}{\left(\frac{\ln b - m_x}{\sigma_x}\right) - \left(\frac{\ln a - m_x}{\sigma_x}\right)}\right] =$$

$$= \frac{-\frac{\partial}{\partial m_x}\left[\left(\frac{\ln b - m_x}{\sigma_x}\right)\right] + \frac{\partial}{\partial m_x}\left[\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]}{\left[\left(\frac{\ln b - m_x}{\sigma_x}\right)\right] + \left[\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]^2},$$

and

$$\frac{\partial}{\partial m_x}\left[\left(\frac{\ln a - m_x}{\sigma_x}\right)\right] = \frac{\partial}{\partial m_x}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\ln a - m_x}{\sigma_x}}\exp\left(-\frac{t^2}{2}\right)dt\right] =$$

$$= \frac{\partial}{\partial m_x}\left[\frac{1}{2} + \frac{1}{\sqrt{2\pi}}\int_{0}^{\frac{\ln a - m_x}{\sigma_x}}\exp\left(-\frac{t^2}{2}\right)dt\right] =$$

$$= -1\sqrt{2\pi}\sigma_x \exp\left\{-\frac{1}{2}\left(\frac{\ln a - m_x}{\sigma_x}\right)^2\right\} = -f(\ln a; m_x, \sigma_x^2), \qquad (30)$$

$$\frac{\partial}{\partial m_x}\left[\left(\frac{\ln b - m_x}{\sigma_x}\right)\right] = -f(\ln b; m_x, \sigma_x^2), \qquad (31)$$

we obtain that

$$\frac{\partial k(a,b)}{\partial m_x} = \frac{f(\ln b; m_x, \sigma_x^2) - f(\ln a; m_x, \sigma_x^2)}{\left[\left(\frac{\ln b - m_x}{\sigma_x}\right)\right] + \left[\left(\frac{\ln a - m_x}{\sigma_x}\right)\right]^2}, \qquad (32)$$

and for the relation (29) we obtain a final form

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$$\frac{\partial \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x} = \frac{f(\ln b; m_x, \sigma_x^2) - f(\ln a; m_x, \sigma_x^2)}{\left[\left(\frac{\ln b - m_x}{\sigma_x} \right) \right] + \left[\left(\frac{\ln a - m_x}{\sigma_x} \right) \right]} + \frac{\ln y - m_x}{\sigma_x^2}.$$

Then, from this last relation, we obtain

$$\frac{\partial^2 \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x^2} =$$

$$-\frac{1}{\sigma_x^2} + \frac{k(a,b)}{\sigma_x^2} \left[(\ln b - m_x) f(\ln b; m_x, \sigma_x^2) - \left(\ln a - m_x \right) f(\ln a; m_x, \sigma_x^2) \right] +$$

$$+ \frac{[f(\ln b; m_x, \sigma_x^2) - f(\ln a; m_x, \sigma_x^2)]^2}{\left[\left(\frac{\ln b - m_x}{\sigma_x} \right) \right] + \left[\left(\frac{\ln a - m_x}{\sigma_x} \right) \right]^2},$$

then when we have in view the relations (30), (31) as well as the following relations

$$\frac{\partial}{\partial m_x} \left[f(\ln a; m_x, \sigma_x^2) \right] = \frac{\ln a - m_x}{\sigma_x^2} f(\ln a; m_x, \sigma_x^2)$$

$$\frac{\partial}{\partial m_x} \left[f(\ln b; m_x, \sigma_x^2) \right] = \frac{\ln b - m_x}{\sigma_x^2} f(\ln b; m_x, \sigma_x^2).$$

Because $\frac{\partial^2 \ln g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)}{\partial m_x^2}$ is independent of the variable y, from (3.3b) we obtain just the Fisher's information (27), if we have in view that $g_{a \leftrightarrow b}(y; m_x, \sigma_x^2)$ is a probability density function. This completes the proof of the theorem.

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